MONTHLY NOTICES

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Professor A. FOWLER, F.R.S., PRESIDENT, in the Chair.

Charles Henry Adamson, Pangbourne Nautical College, Pangbourne;
Amiya Charan Banerji, M.Sc. (Cal.), B.A. (Cantab.), Clare College, Cambridge;
Henry Jamyn Brooks, The Limes, Shalford, Braintree, Essex;
The Rev. J. Maurus Moorat, O.S.B., St. Augustine's College, Ramsgate;
Harry Hemley Plaskett, B.A., Dominion Astrophysical Observatory, Victoria, Canada; and
Arthur Saywell, Smith Street School, Jubilee Street, Mile End Road, E., and 94 Kimberley Gardens, Harringay, N. 4,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Charles Galton Darwin, M.A., M.C., Fellow of Christ's College, Cambridge (proposed by H. F. Newall);
Edwin Powell Hubble, Ph.D., Yerkes Observatory, Williams Bay, Wisconsin, U.S.A. (proposed by H. F. Newall); and
Archibald William Tindall Lucas, Lieutenant, R.G.A. (S.R.), Banker, Berton Hatch, Horsell, Woking (proposed by Herbert Savery).

Thirty-nine presents were announced as having been received since the last Meeting, including, amongst others:—

L. Silberstein, Projective Vector Algebra, presented by the author.

The Spectrum of Nova Aquilæ. By J. Evershed, F.R.S. (Plate 13.)

On June 10 a telegram was received at the Observatory, Kodaikánal, from Mr. C. L. Dundas, I.C.S., of Jhelum, N. India, informing me of his discovery of a nova near Aquila. The star was first seen by him at $3^h\ 30^m$ a.m. Indian Standard Time on the morning of June 9, or 10 p.m. G.M.T. on June 8; and he estimated the magnitude as equal to Altair. Mr. Dundas later gave me interesting details of his observations of the spectrum, which he observed on the night of June 10–11, when the star was estimated as magnitude 0·4. On this date, in addition to the bright hydrogen lines, he noted absorption lines in series, sharp towards the violet. Two nights later the hydrogen lines had developed enormously, and there was "little yellow in the spectrum and not much intervening between the green and the crimson."

The telegram from Mr. Dundas was the first intimation I received of the nova, which was, however, discovered independently about six hours earlier by Mr. G. N. Bower at Madras. He observed it between 9 and 10 p.m. on June 8, or between 3.30 and 4.30 p.m. G.M.T. June 8. This, so far as I know, is the earliest observation of the nova, and Mr. Bower also estimated its magnitude at that time as equal to Altair.

Instruments Employed.

At Kodaikánal, photographic work on the spectrum was attempted immediately, as it was considered of importance to secure records of the earlier stages; the star was extremely well placed for observation, passing within 10 degrees of the zenith at culmination. The first photograph was secured on June 10–11 about midnight, or June 10, $6^h\ 30^m$ G.M.T. The 15-inch Hyderabad lens was used in conjunction with the 18-inch siderostat, and the prism spectrograph built last year for work on Venus. The spectrum is a very imperfect one owing to the steep colour curve of the 15-inch lens in the violet, but it is of interest in showing no defined bright lines or bands but only a continuous spectrum with faint groups of absorption lines. The night of June 11 was wasted in a futile attempt to get a second-order grating spectrum with iron arc comparison, the grating spectrograph being already in adjustment for photographing Venus spectra. By the use of a shorter camera than is used for Venus and an exposure of four hours, a feeble image was obtained showing an ill-defined band corresponding to Hγ emission.

On June 12 the prism spectrograph was rearranged and transformed into a prismatic camera of 6-inch aperture and 7 foot 6 inch focal length. A 6-inch photo-visual lens by Cooke was arranged as camera lens, and the full aperture used of the two excellent 45° prisms acquired from the dismantled Poona Observatory. The prism box, with sand-bag packing to secure uniform temperature, was ready for immediate use; but the camera had to be improvised, and the adjustments for focus and inclination of plate, always troublesome in a slitless instrument, could not be perfected in time for the first photographs exposed on the night of June 12–13. In these the focus is perfect only in the less refrangible region between D and F, but there is much interesting detail visible also in the violet region as far as Hδ.

By the night of June 13–14 the camera had been correctly adjusted and the guiding arrangements also perfected, so that from this date a very interesting series of spectra was secured; these are over 4 inches in length (104 mm.) between Hβ and Hϵ, and at the red end they include Hα photographed on a separate strip of red sensitive plate placed at the end of the slide. On most of the plates a comparison spectrum of Arcturus is impressed, from which the wave-lengths of the nova lines can be determined. On June 20 a small prismatic camera of 2-inch aperture and 2 feet 6 inches focus, with two 60° prisms of ultra-violet glass, was arranged in the beam of light from the 18-inch siderostat. The photographs obtained with this camera are on a scale less than one quarter those of the larger instrument, but they show the ultra-violet region as far as Ho.

As both prismatic cameras were worked simultaneously, the comparison spectrum of Arcturus appears on the small-scale spectra exactly as in those taken with the 6-inch camera.

The guiding arrangements and the method of photographing a comparison spectrum are, I think, new, and as they have proved very effective, a short description may not be out of place. The prismatic cameras were fixed in a horizontal position with the prism edges vertical; it was necessary therefore to give the star a vertical movement in order to broaden the spectrum. This was effected in the following way: the prism box containing the two 6-inch 45° prisms is mounted on a movable carriage running on iron rails passing directly in front of the 12-inch image-forming lens of the spectroheliograph. When the prismatic camera is to be used the prism box is run into position directly in front of this lens, and clamped centrally with respect to the beam of light from the 18-inch siderostat. In this position it obstructs most of the light falling on the lens, but two small segments receive light passing above and below the prisms; these together have an area of 12 square inches, which is quite sufficient to form a bright and well-defined image of a star on the slit plate of the spectroheliograph, 21 feet distant. A mirror was therefore arranged to reflect this image into an eye-piece having a finely ruled glass réseau in its focal plane. By the aid of a plumb line and a pair of lenses the réseau was easily adjusted with its rulings truly vertical and

horizontal. The guiding then consisted in bringing the star to the central vertical line near the top of the field of view and allowing it to drift slowly downward through an interval defined by the horizontal lines, and repeating this operation as many times as needed to secure a fully exposed spectrum photograph. To get a *uniform* downward drift the siderostat axis was put out of adjustment in azimuth by a definite amount; then in guiding the declination slow motion was not touched during the exposure and attention was concentrated on holding the star on the line by means of the R.A. slow-motion motors only, the driving clock being also regulated so that the least amount of deviation should occur.

For the comparison spectrum, it was merely necessary to bring the star whose spectrum it was desired to place beside the nova spectrum to the same central line of the réseau, but in a different part of the field. The exposure was made either before or after the exposure on the nova; but if great accuracy were required, the exposure might have been made both before and after that on the nova, to eliminate the effects of temperature change in the prisms.

The focal length of the 12-inch lens being nearly three times that of the large prismatic camera, and over eight times that of the smaller instrument, errors of guiding are reduced proportionately; so that not only is it possible to secure well-defined spectral images, but also a very accurate alignment of the comparison spectrum, provided no temperature change has occurred in the prisms between the successive exposures.

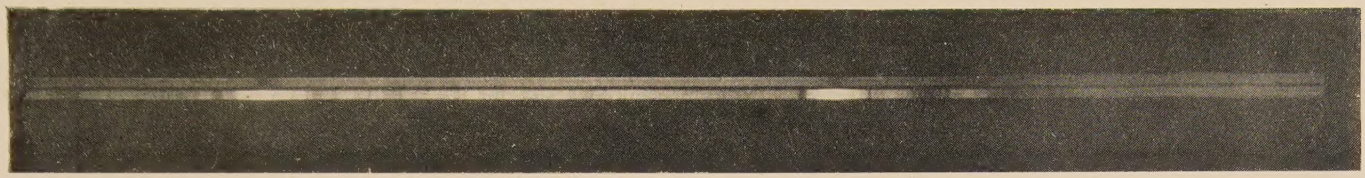
The three spectra reproduced in Plate 13 show the character of the spectra obtained with this apparatus. Fig. 1 is the mixed spectrum of dark lines and bright bands obtained on June 13, $6^h\,37^m$ G.M.T. Fig. 2 is the spectrum of June 19, $4^h\,04^m$ G.M.T., and shows the sharp boundaries of the hydrogen bands, particularly Hβ. Fig. 3 is the first of the series obtained with the 2-inch prismatic camera. The times on the plate are in I.S.T.

Unfortunately for the Kodaikánal results, the seeing was excessively bad throughout the month of June, and this of course set a limit to the definition of the spectrum lines. It was as bad near the zenith as elsewhere, and on many nights the image of Arcturus would constantly expand to a trembling disc 15″ in diameter; and sudden displacements of many seconds of arc, impossible to avoid by anticipation, were even more exasperating. This was not due to irregular driving of the clock, but to refraction changes due to the chaotic state of the air above the mountain—one of the serious drawbacks of a mountain top in a tropical climate which is much disturbed by monsoon winds.

The photographs reveal these irregularities in small distortions of the lines in the Arcturus spectra. In the nova spectra, which are built up by several successive traverses of the star, the lines are widened and diffused by the bad definition.

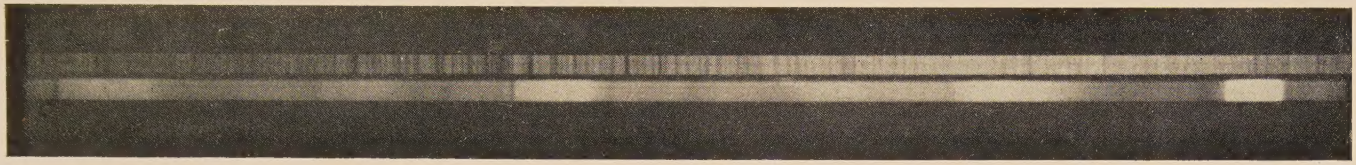
In the work of arranging the apparatus for the nova and

Altair. 1918 June 14, $12^h\ 57^m$ to $1^h\ 25^m$.



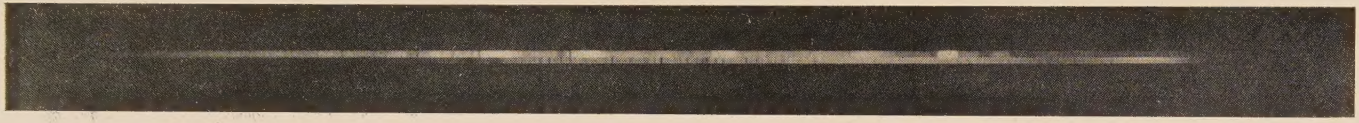
Nova Aquilæ. 1918 June 13–14, $11^h\ 30^m$ to $12^h\ 44^m$.
Fig. 1.

Arcturus. 1918 June 19, $9^h\ 25^m$ to $9^h\ 44^m$.



Nova Aquilæ. 1918 June 19–20, $9^h\ 51^m$ to $0^h\ 28^m$.
Fig. 2.

Nova Aquilæ. 1918 June 18–19, $10^h\ 56^m$ to $0^h\ 57^m$.



Arcturus. 1918 June 18, $10^h\ 32^m$ to $10^h\ 50^m$.
Fig. 3.

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making the exposures I had the very able assistance of the late Mr. R. J. Pocock, Director of the Nizamiah Observatory, who was visiting the Kodaikánal Observatory at the time. The réseau in the guiding eye-piece was his suggestion; and it was due to his very resourceful help that we were able to get all the arrangements perfected by the evening of June 13.

Owing to the failure of the monsoon in Southern India, the weather was exceptionally clear; that is, in the 34 nights from June 8 to July 11 inclusive, about 12 were clear and 6 more or less hazy. Excluding the first two unsuccessful attempts on June 10 and 11, good photographs were secured on 12 nights, and particulars of these are given in the following list:—

Date.	G.M.T. of Mid-exposure.	Duration.	Star.	Remarks.
1918.	h m	m		
June 12	(1) 4 42	60	Nova.	Clear sky, definition poor.
	5 30	15	Arcturus.	
	(2) 6 30	40	Nova.	
	7 07	25	Altair.	,,
13	6 37	74	Nova.	
	7 41	28	Altair.	,,
16	9 49	92	Nova.	Much interruption from hazy cloud, definition atrocious.
	10 54	2	Venus.	
	11 25	41	Nova again.	
18	(1) 3 51	79	Nova.	Definition very bad, image of Arcturus = Venus in size although in zenith.
	4 21	14	Arcturus.	
	(2) 5 11	18	Arcturus.	Nova exposure interrupted by much hazy cloud.
	6 26	121	Nova.	
19	4 04	19	Arcturus.	
	5 39	157	Nova.	
20	4 23	23	Arcturus.	
	5 50	119	Nova	
23	5 33	33	Arcturus.	Clear throughout.
	7 08	135	Nova.	
July 2	4 24	23	Arcturus.	Cloudy, 1 prism only used from this date.
	5 42	116	Nova.	
3	5 06	29	Arcturus.	Sky not clear; exposure interrupted by hazy cloud, definition fair.
	5 56	57	Nova.	
4	10 33	45	Nova only.	Hazy cloud.
6	5 11	38	Arcturus.	Clear throughout, definition fair.
	7 21	148	Nova.	
11	4 01	84	Nova.	Clear throughout.
	5 02	19	Arcturus.	

On June 18 the small prismatic camera was operating, and from this date a double set of photographs was obtained. All the large-scale plates photographed in July were obtained with a single 45° prism instead of the two used in June, when the star was bright enough to admit of the higher dispersion.

Magnitude Observations.

The varying magnitudes of the Nova were estimated by Mrs. Evershed as follows:—

Date. 1918.	G.M.T. h m	Magnitude.	Remarks.
June 10	6 00	0·4	Sky hazy.
10	10 30	0·2	Clear.
11	4 25	0·6	,,
16	11 30	2·1	Sky hazy.
18	3 30	2·0	Clear.
20	6 00	2·1	,,
23	6 30	3·0	,,
July 2	6 30	3·4	Fair.
4	8 30	2·8	Hazy.
6	6 10	3·2	Clear.
9	10 45	3·6	Star low in west.
12	11 02	4·4	,, ,,
15	4 00	4·3	Clear.
15	10 50	4·0	Doubtful owing to low altitude.

The stars used for comparison and their assumed magnitudes were:

Star.	Magnitude.	Star.	Magnitude.
Vega	0·1	ζ Aquilæ	3·0
Altair	0·9	η Serpentis	3·4
θ Scorpii	2·0	λ Aquilæ	3·6
σ Sagittarii	2·1	β Aquilæ	3·9
γ Cygni	2·3	θ Serpentis	4·5
γ Aquilæ	2·8	ω Aquilæ	5·1

Determination of Wave-Lengths.

Of the spectra obtained six were selected for measurement, viz. those photographed on June 12, 13, 19, 20, July 6 and 11. The results are collected in Tables I., II., and III. at the end of this paper. Each table gives the wave-lengths and intensities for two consecutive dates; in Tables I. and II. the intensities are for the absorption lines only, and Table III. for the emission bands only, absorption lines being practically absent in the July plates.

In Table I. the wave-lengths are derived from the position of the hydrogen lines β, γ, and δ in Altair, assuming the wave-lengths to

be 4860·7, 4339·9, and 4101·2 respectively, corresponding to a motion of the star of −49·6 km./sec. towards the Earth. This velocity is the sum of the radial velocity −33 km./sec., and the component of the Earth's orbital motion −16·6 km./sec.

In Table II. the wave-lengths are derived from the numerous known lines in the spectrum of Arcturus. The wave-lengths of these are taken from Rowland's table of solar lines, increased by an amount corresponding with a radial velocity of +17·4 km./sec., Arcturus radial velocity being −5·8 km./sec., and the component of the Earth's velocity on June 20–21 +23·2 km./sec.

In Table III. the wave-lengths are also derived from the spectrum of Arcturus, but the component of the Earth's velocity being greater in July, a correction to the Arcturus lines corresponding with +21 km./sec. has been applied.

The wave-lengths in all the tables are subject to a correction for the component of the Earth's orbital velocity in the direction of the star of −9·8 km./sec. on June 13–14, −6·8 km./sec. on June 20–21, and +2 km./sec. on July 6–11.

The wave-lengths in all cases have been determined by graphical methods, using the numerous known lines in the spectrum of Arcturus to form a dispersion curve. In this work Mr. Sitarama Ayyar has given me valuable assistance. In order to identify with certainty the star lines or groups of lines with solar lines, especially in the difficult region between 4415 and 4860, a good solar spectrum taken with a narrow slit and having nearly the same dispersion as the star spectrum was found to be indispensable.

In the first and third columns in the tables an emission band is distinguished from absorption lines by a vertical line connecting two or more wave-lengths, these marking the approximate limits of the band, or the limits of the region of greatest intensity. These limits are in many cases very ill-defined, especially in the region between Hβ and Hγ. The limits of the hydrogen emission-bands are ill defined on June 13 and 14, but remarkably well defined on all later dates when photographs were obtained, that is, from June 16 to July 11.

The Absorption Spectrum of Hydrogen.

The two plates of June 12 show a double series of hydrogen absorption lines, separated by 10 to 12 angstroms, and displaced towards violet by amounts equivalent to velocities in the line of sight of nearly 1600 km./sec. and 2200 km./sec.

On the 13th the double series is again seen, and the more refrangible set of lines have widened on the side towards violet, Hβ being very definitely bounded and 9 A. in width. On account of this widening, the centres of the lines are still further displaced to the violet. In the next plate, taken on June 16, the more refrangible absorption lines have disappeared, and of the less refrangible series Hα and Hβ are absent, Hγ is faint, and Hδ strong. On the 20th the lines have become indistinctly double,

and on the 23rd the lines γ and δ have narrowed, owing to the disappearance of the more refrangible component. This is the last plate taken in June, and in July even Hδ had almost disappeared. On July 6 a small-scale plate shows Hδ faintly, and almost in the same position with reference to the Arcturus lines as on the 23rd, but its precise wave-length cannot be determined, as the small prismatic camera was not specially protected against temperature change, as was the larger instrument.

The most reliable values of wave-length and velocity have been obtained from the Hδ line, owing to its strength and the higher dispersion compared with Hβ and γ. The wave-lengths and deduced velocities for Hδ on the different dates in June are given below.

Date.	λ.		Km./sec.	
	a.	*b.*	*a.*	*b.*
1918 June 12	4080·2	4071·9	1586	2193
13	4079·6	4068	1630	2478
16	4078·8		1689	
18	4078·1		1740	
19	4077·9		1754	
20	4076·9		1828	
	4078·3		1725	
23	4078·1		1740	

The velocities here shown are subject to a correction for the component of the Earth's orbital velocity, which is − 10 km./sec. on June 12 and decreases to − 5·6 km./sec. on June 23. Measures of the lines β and γ in these plates show that the displacements are approximately proportional to wave-length, but this relation is best seen in the plate of June 19, in which Hϵ can be measured. For this plate I get the following values:—

	Hγ.	Hδ.	Hε.
Normal wave-lengths	4340·6	4101·9	3970·2
Observed wave-lengths	4315·4	4077·9	3947·0
Displacements	25·2	24·0	23·2
Displacement equivalent to 1753 km./sec. .	25·37	23·98	23·21

The greatest discordance here shown is in the Hγ displacement, but this line is the least satisfactory to measure. I have no hesitation in ascribing these large displacements to motion in the line of sight, as no other explanation seems to me in any degree satisfactory, and the movements are not more than two or three times greater than those observed in solar eruptions.

Considerable accuracy has been obtained in the measurements of the δ line, and the velocity determinations are relatively correct to about 5 km./sec. for all the plates after June 13. There may, however, be a small systematic error affecting all the values due to errors in estimating the mean wave-lengths of groups of lines

which appear in the spectrum of Arcturus as single lines. The measures of June 12 and 13 are less accurate, since they are dependent on measures of the broad and ill-defined hydrogen lines in the Altair spectrum.

The results show that there is a steady increase of velocity from June 12 to June 20, when the lines became double. It amounts to 44 km./sec. between June 12 and 13, and averages 21 km./sec. daily from June 13 to 19. There is a further increase of 22 km./sec. on June 20 if the mean velocity on this date is taken. On June 23 the more refrangible component of the double line has disappeared, and the less refrangible component indicates the same velocity as on June 18. Judging by the small-scale plates taken in July, it is probable that very little change took place in the wave-length of Hδ after June 23.

An acceleration of velocity is also recorded under *b* in the table above amounting to 285 km./sec. between June 12 and 13. This refers to the second set of absorption lines which disappeared between June 13 and June 16. The rapid fading of these lines, and the more gradual disappearance of the less refrangible series, suggests that independent masses of relatively cool hydrogen were being ejected from the star and dissipated in space: that is to say, enormous prominences were thrown out and moved with accelerating speed under the influence of a repulsive force. Observations at Kodaikánal have shown that the larger eruptive prominences on the sun move outward with accelerating velocity and become dissipated in space. The largest velocity I have measured in a mass of hydrogen leaving the sun is 457 km./sec. at a height of 6′ above the chromosphere, and the largest velocity deduced from displaced lines at comparatively low altitudes was 360 km./sec. If the hydrogen in a prominence were dense enough to remain visible by absorption at a height of a million miles above the sun, velocities of the order of 1000 km./sec. would certainly be recorded, owing to the acceleration. Fenyi has, in fact, observed velocities in the line of sight as great as 890 km./sec.*

If the motion of a solar prominence is due to light-pressure, much higher velocities are to be expected in the case of a nova, where the emission of light is so prodigiously great in the initial stages. The great reduction of light after the first week or so may account for the tendency of the velocity to become constant after June 23.

In the series of small-scale spectra the absorption lines of hydrogen can be traced as far as Hο on June 20, but they are extremely faint and narrow on July 6. On this date, beginning at Hδ, they can still be traced to H1λ. An interesting feature in the ultra-violet is the absence of any strong emission bands adjoining the absorption lines. Hζ shows more or less emission, but from this line onwards towards the smaller wave-lengths the dark lines stand on a nearly uniform background. The continuous spectrum appears to extend far beyond the theoretical limit of

* *Astronomy and Astrophysics*, **11**, 63.

the series of displaced lines, which would be at 3625. The hydrogen emission is evidently relatively weak in the ultra-violet, while the absorption is weak at the red end of the spectrum. In this the absorption differs from that of the solar chromosphere and prominences, in which Hα is always the strongest line, the absorption decreasing towards the violet, and usually ending at Hε.

The Emission Spectrum of Hydrogen.

The hydrogen bands in the nova are remarkable for their width and sharply defined limits, and for the unchanging wave-length of these limits from June 16 to July 6. The definition of the Hβ band after June 13 is such that its edges may be measured with an error not exceeding ¼ A. I give in Table IV. the results of my measures of the three bands β, γ, δ on the different dates, with the mean positions of their centres; these remained practically constant throughout, and indicate a radial velocity of −83 km./sec. with reference to the Earth, or −78 km./sec. with reference to the Sun, if the displacement may be interpreted as motion. The last plate of the series, taken on July 11, shows a slight change in the position of the violet edges of the bands, which extend about 1½ A. further towards violet, the edges towards red remaining the same. The centres on this date are therefore shifted about 0·7 A. towards violet compared with the earlier plates.

It will be noted that the widths of the bands are very nearly proportional to wave-length, excepting that Hβ is about 2 A. wider than this relation would indicate; but this is easily explained by the great density of the β image. The increased width of the less refrangible bands makes it quite certain that pressure is not concerned in the widening, for according to Rossi* the broadening by pressure of the hydrogen lines varies as the inverse third or fourth power of the wave-length, Hδ widening twice as much as Hβ, whereas in the nova Hδ is four-fifths of the width of Hβ. It seems probable that the widening must be due to motion, but if so, limiting velocities of about 1580 km./sec. in either direction are required. It may be mentioned in this connection that the Hα line in the solar spectrum is about 1 A. in width, and is even more definitely bounded than the α band in the nova. This width is probably a temperature effect, due to molecular movements with limiting velocities of about 22 km./sec.

The internal structure of the hydrogen bands does not vary greatly in my series of spectra. On June 12 and 13 very little of this structure is visible, the intensity increasing towards the centres of Hα and Hβ with fair regularity. Hγ is also strongest in the centre, but the band is broken up by absorption lines superposed upon it. On the dates subsequent to June 13 these absorption lines have vanished, and the negatives show three

* *Astrophysical Journal*, 34, 302.

regions of greatest density in each band. The approximate positions of these maxima are as follows :—

Hβ	4857	4864	4882
Hγ	4337	4342	4356
Hδ	4098	4105	4118

On June 23 the structure is slightly modified by the presence on each emission band of a very obscure absorption line at 4854, 4334, and 4096, and in addition there is a rather marked reduction of density between the two less refrangible maxima in each band. The July plates show very feeble indications of the same three regions of greater density, but only in the ultra-violet, where the bands are not over-exposed. Thus, throughout the series of plates no striking changes have been recorded in the emission bands.

The Calcium Lines H and K.

The absorption line K, displaced nearly 23 A. to violet, is clearly shown in the small-scale spectra of June 18, 19, 20; it is slightly weaker than Hε or Hζ, and on the 23rd becomes weaker still. On July 6 it is very faint and ill-defined on the red edge of the emission band of Hζ. On June 18 there are traces also of an emission band, but this is not seen on the later plates. The calcium H line in these spectra is not separated from Hε.

In the large-scale spectra which include the region, a very narrow and faint absorption line is found superposed on the Hε emission band, and nearly coincident with H in Arcturus. This is seen on the plates of June 16, 19, 20, and 23, and on the plate of the 19th, which extends further in the direction of the ultra-violet: a similar fine line is found almost coincident with K in Arcturus. There are faint traces also shown on some of the plates of an undisplaced calcium line at 4227.

Measures of the line H give the following wave-lengths corrected for the displacement of the Arcturus lines :—

June 16	3968·4
19	3968·3
20	3968·5
23	3968·2
Mean	3968·35

The measures are somewhat uncertain, owing to the weakness of the reference lines in Arcturus in this region, and the mean wave-length may be subject to an error as large as ±0·1 A. The interest of the result lies in the very small displacement shown, which amounts to −0·28 A. compared with Rowland's values of H. This implies a velocity towards Earth of 21 km./sec., or, deducting the Earth's orbital motion of −7 km./sec. at the mean date, there remains a residual radial velocity with reference to the Sun of −14 km./sec.* If the sun is moving in the direction of R.A. 18^h,

* I have probably under estimated the displacement, as, according to Mr. Plaskett's measures, the radial velocity is about −19 km./sec.

Dec. + 30° at − 20 km./sec., the component in the direction of the nova is − 17 km./sec. It follows therefore that the calcium vapour has a very small or zero component of motion in the line of sight with respect to the nearer stars. It probably has no connection with the nova, and may occupy any portion of space between the Earth and the star.

Narrow absorption lines at H and K have been observed in the spectra of other novæ: measures by Campbell and Wright in Nova Persei gave values of velocity with respect to the sun between + 4·2 and + 7·7 km./sec. They also measured fine absorption lines at D, and these gave the value + 3·3 km./sec.;* but the component of the sun's motion in the direction of Nova Persei is appproximately + 4 km./sec.; so here again the calcium appears to have been almost at rest in the line of sight with respect to the sidereal system. In the case of Nova Geminorum No. 2, the fine lines H and K, the D lines, and several others were measured by Adams and Köhlschutter, who found a radial velocity of + 10 km./sec. with reference to the sun.† Here the component of the solar motion is + 9 km./sec., a still closer approach to a zero velocity in the calcium and sodium cloud.‡

The Iron Lines.

The principal enhanced lines of iron are certainly present in the spectrum of June 12 and 13 as a double series of displaced absorption lines, and the less refrangible line of each pair is accompanied on the side towards red by a broad emission band, as in the hydrogen series. In the green region two of these emission bands and their absorption lines were at first taken to be due to the helium lines 4922 and 5016, and the third band nearest the red end to the magnesium triplet *b*. But the absorption lines accompanying this last band are single, not triple, and certainly represent the pFe line 5169. If a correction is applied corresponding with velocities of 2150 km./sec. and 1550 km./sec. to the three pairs of absorption lines, they reduce to the wave-lengths of the three pFe lines, as is shown below.

Observed λλ. (Mean of June 12 and 13.)	Correction.	Corrected λλ.		pFe.
4888·2	+ 35·6	4923·8	4924·1	4924·1
4898·9	+ 25·4	4924·3		
4981·6	+ 36·2	5017·8	5018·2	5018·6
4992·6	+ 25·9	5018·5		
5133·3	+ 37·2	5170·5	5169·7	5169·2
5142·3	+ 26·7	5169·0		

* *Astrophysical Journal*, **14**, 280, 281, and *L.O. Bulletin*, No. 8.

† *Ibid.*, **36**, 308.

‡ I find that Hartmann was led to the same conclusion with reference to δ Orionis and Nova Persei, and there are several binary stars in the Milky Way in which the position of the H and K lines is consistent with the assumption of zero motion with reference to the sidereal system.

If it is assumed, therefore, that the absorbing iron has nearly the same velocity as the hydrogen, the corrected wave-lengths agree, within the limits of accuracy of the measures, with the three enhanced lines of iron in this region.

In the more refrangible region this interpretation is confirmed by the presence of displaced absorption lines and wide emission bands which correspond with the principal pFe lines. The emission bands persist after the absorption lines have disappeared, but become extremely faint in the July plates. The lines and bands identified with pFe are shown in Tables I., II., and III.

The more refrangible components of the pairs of lines die out after June 13, as in the hydrogen series; but the less refrangible components in the green region persist until June 19, and show a diminishing wave-length or increasing velocity, as indicated in the table following.

	λ 4924.	λ 5018.	Mean Velocity km./sec.
June 12	4898·7	4992·2	1563
13	4899·0	4993·0	1528
16	4895·6	4989·5	1738
18	4895·5	4989·5	1741
19	4895·1	4989·2	1762

The progression is not regular, probably owing to the uncertainty of the measures of very weak lines; but the mean velocity indicated increases about 200 km./sec. between June 12 and June 19. The hydrogen δ line gives values increasing from 1586 to 1750 km./sec. for the same dates. The iron vapour therefore shares in the accelerating movement of the hydrogen.

The enhanced lines of iron do not appear to have been recorded as a conspicuous feature in the early stages of previous novæ; but, judging by the comparative tables published by Lockyer,* they seem to be generally present as emission lines in the second or bright-line stage.

The Absorption Band at 4055 – 4067.

This band has undergone a very remarkable series of changes. It appears first on the plate of June 16 as an ill-defined and rather weak absorption band between 4057 and 4067. It is more pronounced on June 18, when two maxima are shown within it. On June 19, the double band is bodily shifted towards violet about 3 A. On June 20 it has returned to its original position; on the 23rd, it has again shifted rather more towards violet. On the next plate, taken on July 2, the band has changed from absorption to emission and occupies about the same position as on June 23. On July 3 it is again a very strong absorption band displaced to violet as before, and on the 4th it is much the same,

* Sir N. Lockyer, *On Some of the Phenomena of New Stars*, Publications of the Solar Physics Committee, 1914.

the spectrum being completely cut in two by the band owing to the entire absence of light at this point. On the 6th it is less dark but still in the same position as on the 4th, and on the 11th it has nearly vanished, showing very faintly as an emission band.

When most shifted towards violet the band occupied the position between 4053 and 4064 and when most shifted towards red it was between 4058 and 4069. This band does not appear to have been a striking feature in the spectrum of Nova Persei.

The Nebular Lines.

There is no distinct evidence of the presence of nebular emission lines before July 2 when the band at 4992–5046 becomes stronger than the band 4895–4950. These two bands in the green represent the enhanced iron lines 4924 and 5018, and possibly the helium lines 4922 and 5016 contribute some of the emission. They are of about equal intensity during June, and the increase of intensity of the less refrangible band in July, I consider is due to the development of the nebular line 5007 superposed on it. On July 11 this band has become very strong, with a maximum of intensity near the violet end at 4979. On this date also a weak band has appeared at 4930–4966, the greatest intensity lying between 4930 and 4946; this probably represents the nebular line 4959, with faint helium (4922) and pFe (4924) emission mixed with it.

Another emission band which became very intense on July 11 forms an extension towards red of the Hγ emission band. This band may possibly represent the helium line 4388, and a nebular line measured by Keeler at 4365; it is very faint on the earlier plates, and on July 11 it is less intense than Hγ. The region of maximum intensity lies between 4378 and 4389, but the band evidently extends towards violet until it becomes merged in the hydrogen band. A new band also developed on July 11 at 4683–4704, probably due to the chromosphere line 4686, and there are traces of what may be helium emission on the red side of this near 4713. The band 4446–4476 in Table III. is probably in part due to the helium line at 4471, and the 4026 helium line is also shown in Table III. by a band from 4020 to 4029. It is evident that helium emission developed in July concurrently with the appearance of the nebular line 5007.

The Dark-Line Spectrum of June 12–13.

Most of the wave-lengths recorded in Table I. have not been determined with greater accuracy than the nearest angstrom unit, partly on account of the ill-defined character of many of the lines, and partly because of the difficulty of reduction, when the only reference lines are the ill-defined hydrogen lines in Altair. Had Arcturus been photographed instead of Altair, much more accurate results could have been obtained. Doubtless a greater relative

accuracy in the positions of the lines could be secured by further work on the plates, but this does not at present seem to be worth while in view of the fact that a more perfect series of plates has been secured elsewhere, covering the same intermediate stage between the dark-line and bright-line spectra. In this mixed spectrum there must always be uncertainty in the case of broad lines as to whether they represent groups of absorption lines or are merely spaces between the emission bands. I believe that the majority of the lines recorded in Table I. are genuine, but their identification is very difficult, owing to the uncertainty as to the displacements. Many have been identified with pFe on the assumption of a double series of lines displaced the same amounts as are the hydrogen lines, this being clearly justified by the results of the more accurate measures made of the pFe lines in the green, where they are conspicuous. On the assumption of a single series, displaced from 23 to 24 angstroms, towards violet, many other lines coincide with the stronger pTi lines given in Lockyer's tables. The pMg line 4481·6, recorded as a conspicuous line by Pickering in Nova Persei, is probably the line at 4460 on June 12 and 4458 on June 13.

The presence of many enhanced lines in the intermediate spectrum, and the strength of the ultra-violet continuous spectrum beyond the hydrogen series, in the bright-line stage, indicates relatively high temperature. My spectra are also extremely weak in the green in relation to Arcturus, but the two plates of the intermediate stage appear to be about equal in intensity to the Altair spectrum throughout the range photographed, that is, from Hα to Hδ.

Concluding Remarks.

The most interesting result deduced from this series of spectra is the changing wave-lengths of the hydrogen lines and of the enhanced lines of iron, indicating increasing velocities of the dissipating gases. The analogy with the solar eruptive prominences which move with accelerating speed is striking; but the differences should also be emphasised. In the solar eruptions the enhanced lines of iron are confined to the chromosphere region, and are never seen high above the Sun's limb; also they never show the large displacements commonly displayed by the hydrogen and calcium lines; the iron vapour does not share in the rapid movements of the hydrogen and calcium. But in Nova Aquilæ the pFe lines give almost the same motion displacements as hydrogen and calcium, and the same acceleration.

Another difference is in the absorption effects. In the Sun, only the denser low-lying prominences give any indication of absorption when seen against the photospheric background, and this absorption is almost confined to the lines α and β. The chromosphere itself gives absorption lines diminishing in intensity from Hα to Hϵ, the rest of the series not appearing at all in the solar spectrum. In Nova Aquilæ, Hδ, ϵ, and ζ appear as the

strongest absorption lines, the intensity diminishing both in the lines towards red and those towards the ultra-violet.

If we may designate the nova eruptions as "prominences," it is evidently necessary to modify our ideas derived from their solar analogues. In the first place, the nova prominences are not isolated streams of ejected gas, as in the sun, but vast masses moving outward in all directions from the central star. If the nova originally possessed a chromospheric envelope, this must have been thrown bodily outward, the moving masses subsequently coming under the influence of the intense light pressure, causing an acceleration of the motion (see note below).

The inference from the absorption phenomena is simply that the nova prominences must be both hotter and denser than the solar prominences.

Notwithstanding the differences mentioned, it seems probable that the forces concerned in the eruption, accelerating movement, and final dissipation of a solar prominence are also operating in novæ, and are largely concerned in the striking but very transient phenomena of the displaced absorption lines.

The measures of the widths of the emission bands after the bright-line stage was reached indicates that the widening is also a Doppler effect. If this is granted, the positions of the edges of $H\gamma$ and δ, which were not widened by over-exposure, indicate an approaching velocity of about 1660 km./sec. for the violet edges, and a receding velocity of 1500 km./sec. for the red edges of the bands, the centres being displaced to violet by an amount corresponding to a radial velocity of about -80 km./sec. The velocity with reference to the star itself would then be 1580 km./sec. This can scarcely be due to a rotation of the star, because in that case there would be a marked diminution of intensity towards the edges of the bands, whereas they have a nearly uniform background intensity from edge to edge, but with three brighter regions unsymmetrically superposed.

If the star in the later stages possesses no condensed opaque nucleus, but consists of a truly gaseous mass, mainly of hydrogen, and expanding uniformly in all directions with a velocity of 1580 km./sec., then the gas on the side towards Earth would not obstruct the radiation from the opposite side, because owing to the change of wave-length it would be transparent to the radiation from the receding gas. The result would be wide emission bands with edges defined by the velocity of the expansion. As the width of the bands is shown to be no indication of pressure, an exceedingly low pressure and low density may be assumed.

It is probably of significance that the lower limit of velocity given by the absorption lines is so near to the upper limit given by the emission bands; for it may be supposed that the absorbing hydrogen in the early stages also possessed this initial velocity of expansion, which was subsequently increased by light-pressure, or whatever the force may be which drives solar prominences outward into space.

Contrasting with these violent movements of hydrogen in the nova, the very tenuous calcium vapour giving the fine absorption lines H and K appears to be in a state of serene immobility, not with reference to Earth or Sun, but stationary in space with respect to the sidereal system.

Kodaikánal:
1918 *December* 4.

Note added 1919 *April* 28.

Mr. Plaskett has pointed out to me that "the great range in the radial components of these outward rushing gases coming from one hemisphere of the star should make the lines very broad and indistinguishable, instead of, as they frequently are, sharp and narrow."

My reply to this is that the absorption lines may be supposed to be due to gases which have become separated from the main body, like solar eruptions, but on a vastly greater scale. It is easy to show that an outer shell of absorbing gas, separated from the star by a distance equal to the star's radius, would give absorption lines of such width that the minimum velocity indicated by the less refrangible edges would be 87 per cent. of the maximum, represented by the more refrangible edges. Gases moving out at angles greater than 30° to the line of sight would not occult any part of the star, and could not be seen by absorption. For velocities of 1600 and 2400 km./sec. the widths at Hγ would be 3 A. and 4·6 A. respectively, which is comparable with the actual widths of the hydrogen lines on June 13, although the metallic lines were narrower. The more refrangible line Hγ was, in fact, nearly twice the width indicated.

According to Mr. Plaskett's description of the spectral changes, it appears that both the hydrogen and the metallic lines were broader and more diffuse on June 10 than on later dates, which would indicate a smaller separation of the absorbing gases from the main body during the early stages.

Professor Newall considers the more refrangible series of hydrogen lines on June 13 to be blends, for each of the lines δ, γ, β on June 15 became two distinctly separated lines. This shows that two independent masses of hydrogen, moving with different velocities, were concerned in the widening on June 13.

It seems probable that only the gases of the star's original chromosphere lying outside the intensely luminous photosphere would be driven out in the way indicated if the repulsive force is light pressure.

TABLE I.

Spectrum of Nova Aquilæ.

June 12, 6h 30m G.M.T.		June 13, 6h 37m G.M.T.		Remarks.
λ.	I.	λ.	I.	
4071·9	10	4068	?	Hδ 2nd absorption line.
4080·2	8	4079·6	8	Hδ 1st absorption line.
4081 } 4119 }	...	4079 } 4118 }	...	Hδ emission band with maxima at about 4095 and 4105.
4141	6	4141	5	pTi 4163·8.
4150	7	4150	6	} pFe 4179·0, also pTi 4172·0.
4156	5	4157	4	
		4159 } 4174 } 4191 }	...	Faint emission band visible also on the 12th but not measured. Maximum intensity 4174.
4192	2	...	...	Probably present, but very faint on 13th. ? pSr 4215·7.
4197	4			
4200	2	4199	2	
4203	1	4205	2	} pFe 4233·3.
4210	7	4211	8	
4224	3	4225	2	
		4227 } 4235 }		Emission band visible also on 12th.
4267	4	4269	3	pTi 4290·3.
4271	4	4272	2	pFe 4302·3, 4303·3. pTi 4294·2.
		4276	...	Apparent emission line on edge of absorption line.
4277	3	4278	3	pTi 4300·2.
4280	2	4279	2	Ill defined. pFe 4302·3, 4303·3.
4285	3	4286	0	pTi 4308·1.
4291	4	!		pTi 4313·0 and 4315·1.
4299	2	4299	0	
4303	2			
4308	7	4307	12	Hγ 2nd absorption line. The line at 4303 on the 12th is merged in Hγ absorption on the 13th.
4318	4	4319	5	Hγ 1st absorption line and pTi 4338·0.
4319		4320		V limit of emission band.
4328	2	4330	2	
4346	2	4346	2	Hγ emission band with many absorption lines superposed on it. Maximum intensity at 4332 to 4341. pFe and ? pTi lines.
4351	3	4351	1	
4361	3	4360	2	
4374	6	4373	5	
4376		4376		R limit of emission band.

TABLE I.—*continued.*

June 12, $6^h\ 30^m$ G.M.T.		June 13, $6^h\ 37^m$ G.M.T.		Remarks.
λ.	I.	λ.	I.	
4380	2			
4384	0			
4396	2	4394	2	Broad line. ? pTi 4417·8.
4423	4	4421	3	Well defined. pTi 4443·9.
		4432	2	Ill defined.
4448	3	4446	4	pTi 4468·6.
4451	3	Absent.		
4456	1	Absent.		
4460	4	4458	2	pMg 4481·3.
4467	2	4464	1	Ill defined. pTi 4488·4.
4480	2	4478	2	pFe 4508·4 and pTi 4501·4.
4487	0	4485	1	
4489	0	4490	1	Group of lines pFe 4520·4, 4522·7. Apparent emission band between 4500 and 4510.
4503	0	4498	1	
4513	2	4508	1	
4517	0	?		pFe 4549·6 and pTi 4549·8.
4527	3	4525	2	
4532	0	?		Traces only of these lines on June 13, pFe 4556·0.
4538	0	?		
4542	1	?		
4550	3	4548	3	pTi 4572·1.
Absent	...	4553	2	pFe 4584·0.
4561	1	4557	2	
4563	0	Absent.		
?	...	4571	1	Wide lines very faintly indicated on June 12.
?	...	4579	1	
Absent	...	4586	2	
4595	2	4590	1	
		4600	1	
4607	1	4610	3	Wide and diffuse.
Absent	...	4618	1	
?	...	4633	...	Emission band barely seen on June 12.
?	...	4650	...	Emission band barely seen on June 12. There is evidence of emission between all the absorption lines from 4500 to 4650.
4737	0	?		Traces of this faint group also visible on June 13.
4760	0			
4781	1	4784	0	pTi 4805·2.

TABLE I.—*continued.*

June 12, 6h 30m G.M.T.		June 13, 6h 37m G.M.T.		Remarks.
λ.	I.	λ.	I.	
4799	0			
4824	10	4822	15	Hβ 2nd absorption line, width 9 Å. on 13th.
4836	2	4836	1	Hβ 1st absorption line.
4835	...	4837	...	Hβ emission band, extends faintly from 4825 to 4898 in both spectra, maxima at about 4861.
4886		4886		
4888	4	4888	1	pFe 4924·1.
4899	8	4899	8	
4902	...	4901	...	Emission band, the densest part is from 4918 to 4930.
4919		4918		
4920		...		
4932		4930		
4934		4936		
4949		4949		
4983	8	4984	5	pFe 5018·6.
4993	8	4995	7	
4996	...	4997	...	Emission band with three regions of greater intensity.
5004		5002		
5013		5014		
5026		5028		
5030		5034		
5043		5048		
5133	8	5134	4	pFe 5169·2.
5144	8	5144	5	
5146	...	5148	...	Emission band with three regions of greater intensity.
5153		5156		
5165		5164		
5175		5178		
5186		5185		
5196		5194		

TABLE II.

Spectrum of Nova Aquilæ.

June 19, 5h 39m G.M.T.		June 20, 5h 50m G.M.T.		Remarks.
λ.	I.	λ.	I.	
3912	10?			Plate much under-exposed here.
3933	0			K. Very narrow line.
3947	15			Hε absorption.

TABLE II.—*continued.*

June 19, 5h 39m G.M.T.		June 20. 5h 50m G.M.T.		Remarks.
λ.	I.	λ.	I.	
3948				
3968	0	?		H. Very narrow line superposed on Hε emission.
3979		3979		
3988	...	3988	...	Brightest part of Hε emission on 20th.
4021		4025		Faint emission band, strongest at 4052 to 4055 on 19th, and 4055 to 4057 on 20th.
4052	...	4030	...	
4055		4055		
		4057		
4055	8	4058	8	Broad absorption band slightly marked maxima at 4057 and 4064 on 19th, 4060 and 4067 on 20th.
4067		4069		
4077·9	10	4076·9	4	Hδ absorption, double on 20th.
		4078·3	6	
4079		4079		Hδ emission band with maxima at 4098, 4105, and 4118.
4123	...	4122	...	
4208	1	?		
4210		4210		Faint emission band with undisplaced Ca absorption line superposed. pFe 4233.
4227	0	?		
4256		4254		
4278	0	?		Broad and ill-defined line.
		4314·3	2	Hγ absorption double on 20th.
4315·4	8	4315·7	4	
4316		4316		Hγ emission with maxima at 4337, 4342, and 4356.
4363	...	4363	...	
4375	0	4375	1	Broad and ill-defined line, or space between emission bands.
		4377		
4378		4378		Faint emission band, maxima at 4416.
4424	..	4424	...	
4443		4443		Very faint emission band.
4474	...	4474	...	
4504		4500		Faint emission band, limits very ill defined, pFe, several lines.
4545	...	4552	...	
4555	1	Absent.		
4587	1	4589	1	Very broad and ill-defined lines.
4592	2	4597	2	
4601	3	4604	3	
4606		4608		Emission band with maxima at 4644. Intensity gradually reduced towards red.
4719	...	4705	...	

TABLE II.—*continued.*

June 19, 5h 39m G.M.T.		June 20, 5h 50m G.M.T.		Remarks.
λ.	I.	λ.	I.	
4810 } 4823 }		4817		Faint absorption band.
		4831	2	Hβ absorption, more refrangible component of double.
4833 } 4887 }		4834 } 4886 }	...	Hβ emission, maxima at 4857, 4864, and 4882.
4895·1	1	?		Probably present, but very faint on 20th. pFe 4924.
4895 } 4950 }	...	4897 } 4947 }	...	Emission band very definitely bounded. pFe 4924.
4989·2	1	?		Probably present on 20th, but plate under-exposed. pFe 5018.
4990 } 5040 }	...	4992 } 5046 }	...	Emission band definitely bounded. pFe 5018.
5147 } 5200 }	...	5162 } 5197 }	...	Emission band, both plates much under-exposed here. pFe 5169.
(6563)		(6563)		Hα emission band well defined and similar to Hβ in structure.

TABLE III.

Spectrum of Nova Aquilæ.

July 6, 7h 20m G.M.T.		July 11, 4h 01m G.M.T.		Remarks
λ.	I.	λ.	I.	
3871	...		...	Hξ absorption line.
3872 } 3898 } 3912 }	3	3872 } 3898 } 3911 }	3	Hξ emission, trace of absorption line at 3898.
3950 } 3990 }	5	3948 } 3989 }	5	Hε emission, maxima at 3974 on 6th, 3973 and 3986 on 11th.
4005 } 4010 }	1	4004 } 4014 }	1	
4023 } 4030 }	2	4020 } 4029 }	0	He.
4034 } 4051 }	2	4038 } 4042 }	0	
4053 4064	...	4053 } 4060 }	3	Strong absorption band on 6th, emission band on 11th.
4066 } 4077 }	2	?		Probably present on 11th.

TABLE III.—*continued.*

July 6, 7^h 20^m G.M.T.		July 11, 4^h 01^m G.M.T.		Remarks.
λ.	I.	λ.	I.	
4079	...	4078	...	Hδ absorption line feebly indicated.
4080–4123	10	4075–4125	15	Hδ emission, maxima at 4105 on 6th, 4116 and 4360 on 11th.
4127–4142	1	Absent.		
4317–4363	15	4315–4366	20	Hγ emission, maxima at 4344 on 6th, 4343 and 4360 on 11th.
4373–4383	3	4378–4389	8	Neb. and He.
4407–4417	2	4396–4403	0	
4432–4435	0	4430–4440	1	
4446–4465	2	4446–4476	3	He?
4480–4488	2	4482–4496	2	
4508	2			
4512–4548	3	4513–4532	1	
		4544–4552	0	
4583–4625	15	4606–4662	20	
4663–4690	4	4683–4704	4	Probably due to the chromosphere line 4686.
4834–4887	20	4832–4887	30	Hβ emission, maxima at 4863 on 6th, too dense on 11th to see details.
4899	1	4930	2	Neb. and He.
4949	0	4946	1	Neb. and He.
4964		4966		
4980–5030	2	4976–5033	8	Neb. and He., maximum at 4979 on 11th, uniform on 6th.

TABLE IV.

Hydrogen Emission Bands.

Hβ.

	Violet edge.	Red edge.	Centre.
June 18	4833·4	4886·7	4860·1
19	4833·2	4886·7	4860·0
20	4833·7	4886·6	4860·1
23	4833·7	4886·5	4860·1
July 6	4834·0	4886·8	4860·4
Means	4833·6	4886·7	4860·1
Mean width, and displacement of centre.	53·1 A.		1·4 A. = −86 km./sec.

Hγ.

	Violet edge.	Red edge.	Centre.
June 18	4317	4362	4339·5
19	4316	4363	4339·5
20	4316	4363	4339·5
23	4317	4360	4338·5
July 6	4317	4363	4340·0
Means	4316·6	4362·2	4339·4
Mean width, and displacement of centre.	45·6 A.		1·2 A. = −83 km./sec.

Hδ.

	Violet edge.	Red edge.	Centre.
June 18	4079	4122	4100·5
19	4079	4123	4101·0
20	4079	4122	4100·5
23	4079	4122	4100·5
July 6	4080	4123	4101·5
Means	4079·2	4122·4	4100·8
Mean width, and displacement of centre.	43·2 A.		1·1 A. = −81 km./sec.

The Spectrum of Nova Aquilæ, 1918 *July* 29.

By the Rev. A. L. Cortie, S.J.

A good plate of the spectrum of Nova Aquilæ was obtained with the Thorp 4-inch prismatic camera on July 29, the time of the trail being from 10.26 to 11.5 p.m. G.M.T. As many of the characteristics of the later stages of the spectrum of a nova appear on this photograph, it is proposed to discuss it in some detail. A reproduction of this spectrum has already appeared in my first paper on the spectrum of the nova, *Monthly Notices*, **79**, 122 *et seq.*, Plate 3, as fig. 3, to which reference can be made.

The magnitude of the star on July 29, according to Dr. Rambaut's measures, was 3·89, and according to Father Hagen, 3·8. On June 15 the star's magnitude was 1·48, so that it had fallen 2·41 magnitudes in that interval and 4·3 magnitudes since June 9. The spectrum has been measured between wave-lengths 3836 and 5028, which extend over 19·37 mm. on the plate. The spectrum, however, extends faintly as far as H_λ in the violet.

The following is a list of the bright bands which were most prominent in the spectrum of the nova on this date, the mean wave-length of the centres of the bands being given. These bands are compared with the corresponding dark lines in the spectra of γ Orionis and α Cygni, and in each case the intensities of the bands and lines are given on a scale 10 to 1. The wave-lengths and intensities of the dark lines in the spectra of γ Orionis and α Cygni have been taken from the *Catalogue of Four Hundred and Seventy of the Brighter Stars*, published by the Solar Physics Committee in 1902.

In the more extended table which follows, the first column contains the wave-lengths of all the measurements made on the plate. The settings of the micrometer wire have been made on the edges and the maxima in the bright bands, and each reading of wave-length is the mean of at least three settings. The next two columns contain the deduced mean wave-lengths and intensities of the various bands. Succeeding columns contain the corresponding wave-lengths and intensities of the dark lines in γ Orionis and α Cygni. There is also a column of chemical origins, and one of remarks on the lines. At the March meeting of the Society, Lieut.-Col. Stratton, D.S.O., read a paper on the spectrum of Nova Geminorum II., 1912, in which he arrived at the important conclusion that a spectrum matched by the helium star γ Orionis, succeeded that which is matched by α Cygni. Hence the reason for including the spectra of both stars in the tables in this paper.

TABLE I.

List of Lines prominent in Spectrum of Nova Aquilæ, 1918 *July* 29.

Wave-length.	Intensity. July 29.	Intensity. June 15.	γ Orionis. Wave-length.	γ Orionis. Intensity.	α Cygni. Wave-length.	α Cygni. Intensity.	Origin.
3889	5	2	3889	10	3889	10	Hζ and He.
3970	6	8	3970	10	3970	10	Hε.
4009	4	2	4009	6	4009	1	O star line.
26	4	...	26	8	26	1	He. Nebula.
63	5	3	...	...	...	...	O star line.
4102	9	8	4102	10	4102	10	Hδ.
4341	10	10	4341	10	4341	10	Hγ.
63	10	8	...	...	...	...	$H_{\gamma 1}$ and Nebula.
88	9	2	86	6	88	1	He.
4470	4	2	4472	8	4472	2	He.
4582	5	3	...	...	4584	7	Fe spark line.
4640	8	3	4641	1	4641	<1	O ? 4637 Nebula.
76	4	...	76	1	...	...	O.
86	4	...	...	...	...	...	Nebula.
4862	9	10	4862	10	4862	10	Hβ.
4922	2	3	4922	5	4924	5	He, or Fe spark line.
57	2	...	...	...	...	...	Nebula.
5006	4	...	...	...	...	...	Nebula.

This list shows that, except for the nebular lines and except for the enhanced iron line 4584 which belongs to an α Cygni, and the oxygen line 4676 which belongs to a γ Orionis spectrum, the other prominent lines are found in spectra representative of both types of stars. But if, in the first place, the relative intensities of the bright lines be compared in the spectrum of the nova on July 29 and June 15, and, in the second place, the relative intensities of the corresponding dark lines in the spectra of γ Orionis and α Cygni be also compared, it will be seen that the spectrum of the nova on July 29, as represented by these more prominent lines, excepting always the nebular lines, approximated rather to that of γ Orionis than to that of α Cygni.

But though the γ Orionis type of spectrum was predominant in those lines of the spectrum of the nova that had become brighter between the dates June 15 and July 29, yet the following extended table will show that in the fainter radiations the α Cygni spectrum was responsible for most of the bright bands. That is, the α Cygni type of spectrum persists, even when the well-marked nebular lines are in evidence, although the γ Orionis spectrum is predominant. The latter type of spectrum does not displace, but is concomitant with, the former.

TABLE II.

Bright Bands in the Spectrum of Nova Aquilæ, 1918 *July* 29.

Wave-length.	Mean Wave-length.	Intensity.	γ Orionis.			α Cygni.			Remarks.
			Wave-length.	Intensity.	Origin.	Wave-length.	Intensity.	Origin.	
3836		3	3836	10	H	3836	8	H	H_{η}.
53		3	54	1	Si *	54	3	Si *	
72		4							$H\zeta_2$.
89	3889	5	89	10	H	89	10	H	$H\zeta$.
3904		4							$H\zeta_1$.
23		0				3923	0	Fe	Very faint lines.
31		0	3930	1		32	1	Ti *	
40		0				39	3	Fe *	
47	3952	4							$H\epsilon_2$.
57									
65	3970	6	70	10	H	70	10		$H\epsilon$.
75									
81	3985	5							$H\epsilon_1$.
88									
96		1	95	4	N	97	1	V*	Narrow band.
4009		4	4009	6	He ?	4009	1	He ?	Centre of diffuse band.
28		4	26	8	He	26	1	He	Centre of diffuse band.
39		1	39	1		38	1	Cr *	Faint diffuse band.
53	4057	5				58	1	Fe *	Centre of diffuse band.
62									4063 Wolf-Rayet stars.
73		1	72	2	O				
80	4038	8							$H\delta_2$.
86									
4100	4102	9	4102	10	H	4102	10	H	$H\delta$.
05									
12	4118	8							$H\delta_1$.
23									
30									
38	4143	1	44	7	He	44	2	He	
48									
55									
64									
74	4170	1	69	3	He	70	0	Cr *	Nebular line 4167.
77									
8		1				85	0	Fe	
91		1	90	2	O	92	1	Ni *	
99						99	1	Fe	

TABLE II.—*continued.*

Wave-length.	Mean Wave-length.	Intensity.	γ Orionis.			α Cygni.			Remarks.
			Wave-length.	Intensity.	Origin.	Wave length.	Intensity.	Origin.	
4206		1	4205	1		4205	1	V *	
16		1				16	2	Sr *	
21		1	22	1		22	0		
31		1				31	1	V *	
43	4261	1							Edge
55			54	1	O	55	2	Cr	Maximum — Probably the nebular line 4265.
70			71	1		70	2	Cr *	Maximum — Probably the nebular line 4265.
78									Edge — Probably the nebular line 4265.
85	4295	1	85	1		85	2	Cr *	Maximum.
93						94	4	Ti *	Maximum.
4304						4304	5	Fe *	Edge, faint.
09		1				08	4	Ti *	Faint line.
17	4322	9							H_{γ_2}.
24									
38	4341	10	4341	10	H	41	10	H	H_{γ}.
44									
56	4359	10							H_{γ_1}.
62									Nebular line 4363.
73		9	88	6	He	88	1	He	Edges of uniform bright
85									band.
95		3	96	2	O	95	5	Ti * Mg *	Centres of three hazy
4410		3	4411	1	Ti *	4411	1	Ti *	bands.
27		3	27	1					
47	4470	4							Edge. Maximum.
66		4	72	8	He	72	2	He	Maximum.
82		4							Maximum.
93		3							Edge. Triple structure
4507	4530	2							Edge.
11		3	4531	1	N	4530	1	Ti *	Maximum.
28		3							Maximum.
42		3							Edge. Triple structure
54		4	53	3	Si *	53	0	Si *	Edge.
67	4582	4							Maximum.
83		5				84	7	Fe *	Edge. Triple structure
96		4							
4605		6	4602 ?	1	N				

TABLE II.—*continued.*

Wave-length.	Mean Wave-length.	Intensity.	γ Orionis.			α Cygni.			Remarks.
			Wave-length.	Intensity.	Origin.	Wave-length.	Intensity.	Origin.	
4617		8							Maximum } Nebular line
34		8	4641	1	O				Maximum } 4637.
56	4640	8	44	1	N				Maximum.
64		7							Edge. Triple structure.
76		4	76	1	O				Double line.
86		4							Nebular line 4686.
97		2							Broad line.
4708	4718	2	4713?	4	He				The band covers the position of the He line.
27									
47		1							Nebular line 4744.
83		0							Very faint.
94		0							Very faint.
4814		1							
40		9							$H\beta_2$.
62	4862	9	4862	10	H	4862	10	H	$H\beta$.
81		9							$H\beta_1$.
4900		2	4922	5	He	4924	5	Fe *	
22									
30		2							
4959.57		2							Nebular line 4959.
80		4							Nebular line 5007.
5006									
28									

In the above table the wave-lengths, which seemingly belong to one and the same bright band, are bound together by a vinculum, the positions of the edges and bright maxima being given. In the columns devoted to the origins of the lines, an asterisk denotes that the line is an enhanced or spark line.

A careful comparison of the wave-lengths of the fainter bright bands with those of the spectrum of June 15 (*Monthly Notices*, *R.A.S.* 79, 174–176) shows that the spectra are practically identical. Hence the conclusion arrived at that, besides the γ Orionis spectrum of the more prominent bands, the fainter bright bands still maintain the character of an α Cygni spectrum.

In *Monthly Notices*, *R.A.S.*, 73, 648 *et seq.*, there is a table of wave-lengths of Nova Geminorum II. which is the result of my measures of a weak photograph of the star taken on 1912 April 24. Dr. Rambaut's magnitude for the star on that date was 6·8. The photograph was taken on an isochromatic plate, with the Hilger compound prism spectroscope, attached to the 15-inch equatorial.

Besides the difference of instrument, the wave-lengths were deduced from a different curve, with different readings for the fiducial points from those employed for Nova Aquilæ. Within the region covered by both plates, that of 1912 April 24 and of 1918 July 29, there are 75 readings of wave-lengths on the latter plate. Within a limit of two Ångstrom units, 28 readings on the Nova Geminorum plate are coincident with those on the Nova Aquilæ plate, and there is a close similarity between the bands measured on the two plates. The comparison shows that the two spectra are practically identical.

The Hydrogen Bands.

An inspection of the above table will show that several of the bright bands have a triple structure. This structure is particularly marked in the hydrogen bands. The H_δ and H_ϵ bands may be taken as examples. They consist of three diffuse bands, separated by dark spaces, the central band being the brightest, and the terminal bands being of about the same intensity. In H_γ, however, the less refrangible of the three constituent bands of the group is as bright as the central band, and is the brightest radiation in the spectrum. The reason of this is that the less refrangible of the three hydrogen bands is coincident in position with the nebular line $\lambda 4363$. In the table the readings are given of the extreme edges of these bands, and also of the centres of the three constituent bands, except in the case of H_ζ, which is more difficult to measure, and of H_β, where the three constituents almost coalesce, in which cases the wave-lengths of the centre of the three bands only are given. The widths of the bands are progressively greater from the violet to the red, which was the case also on the plate of June 15.

Width of Hydrogen Bands.

		July 29.	June 15.
H_ϵ	$\lambda\begin{matrix}3947\\3988\end{matrix}\}$	41	40
H_δ	$\lambda\begin{matrix}4080\\4123\end{matrix}\}$	43	42
H_γ	$\lambda\begin{matrix}4317\\4362\end{matrix}\}$	45	50
H_β	$\lambda\begin{matrix}4834\\4888\end{matrix}\}$	54	59

This progression in the widths of the bands suggests a possible line-of-sight velocity effect, which would require that a composite red band should be wider than more refrangible bands, and in the order of refrangibility. Accordingly, measurements were made of the centres of the components of the triple bands. It was

found that a normal hydrogen spectrum accurately fitted the centres of the central bands of each triplet. The centre of the central bright bands of the H_γ triplet was taken as the fiducial point in the measurements for wave-length.

TABLE III.

Line-of-sight Velocities of the Hydrogen Bright Lines. Nova Aquilæ, 1918 *July* 29.

	λ_1.	λ_1.	$d\lambda_1$.	km./sec.	λ_2.	$d\lambda_2$.	km./sec.
$H\beta$	4862	4881	19	+1173	4840	22	−1356
H_γ	4341	4358	17	+1174	4322	19	−1312
$H\delta$	4102	4118	16	+1170	4083	19	−1389
$H\epsilon$	3970	3985	15	+1133	3952	18	−1360
$H\zeta$	3889	3904	15	+1157	3872	17	−1311
	Means			+1161			−1346

This result, and there seems little reason to doubt its accuracy, would greatly modify a statement made in my last paper (*loc. cit.*, p. 173) to the effect that "the breadth of the bright hydrogen lines does not seem to represent a displacement, but rather a very great symmetrical widening on both sides of the central hydrogen line."

The triple bands characteristic of this spectrum, the central bands representing a stationary source of light and the others a spectrum of recession and approach, would suggest the presence of three sources of light. But this question can only be adequately solved by spectra taken with a high dispersion and a slit spectroscope.

Summary.

1. Measures of wave-lengths of the bright bands, in the spectrum of Nova Aquilæ, 1918 July 29, are compared with the wave-lengths of the dark lines in the stars γ Orionis and α Cygni.

2. Excluding the well-marked nebular bands, the more prominent bands, chiefly helium, agree in intensity rather with γ Orionis than with α Cygni.

3. The fainter bands belong to the α Cygni type of spectrum.

4. Several of the bands, particularly those of hydrogen, are triplets.

5. There is evidence to show that, besides a stationary spectrum, there is a spectrum indicating a line-of-sight motion of recession of about 1160 km./sec., and a similar spectrum indicating a line-of-sight motion of approach of about 1340 km./sec.

Stonyhurst College Observatory:
1919 *May* 5.

Variations in the Ultra-Violet Spectrum of Nova Aquilæ III. (1918). By C. P. Butler, A.R.C.Sc. (Plate 14.)

(*Communicated by the Director of the Solar Physics Observatory, Cambridge.*)

Among the conspicuous changes shown in the spectrum of Nova Aquilæ III. (1918) on photographs taken by Parkhurst at Green River, Wyoming, on the evenings of 1918 June 8, 9, and 10, the marked predominance of three absorption groups in the ultra-violet attracts attention. These groupings suggested the presence of prominent features shown in the solar spectrum when taken with small dispersion, but in the absence of comparison spectra the identification of the detailed structure is somewhat uncertain (*Jour. B.A.A.*, **29**, 56, 1918).

After the spectra of the nova on the three evenings were arranged in alignment, it became evident that by assuming that the absorption structure was affected with the same displacement to the violet as the dark hydrogen series which constitutes the predominant feature of the spectrum on June 8, the main details of the ultra-violet groupings could then be satisfactorily matched with the outstanding groups in small-scale solar spectra.

These groupings are very clearly shown in the small dispersion spectra in *An Atlas of Representative Stellar Spectra*, by Sir William and Lady Huggins, plate ii., especially in the spectra of Saturn, Venus, and Arcturus. Copies were made from these, and mounted in conjunction with the nova spectrum. It was then possible to get a fair approximation to the position of the groups by comparison with Rowland's map.

A short note describing this result was sent to Professor Frost, and he and Professor Parkhurst have very kindly permitted the use of their spectra for illustration.

On June 8 the absorption spectrum apparently consists mainly of the hydrogen series, with calcium H and K, and a strong line near the position of λ4481, the strong spark line of magnesium. On June 9 there are intensifications of the absorption in several places, but specially in three groups in the ultra-violet marked A, B, C on the accompanying illustration, near the approximate positions:—

A = λ3550–λ3585.

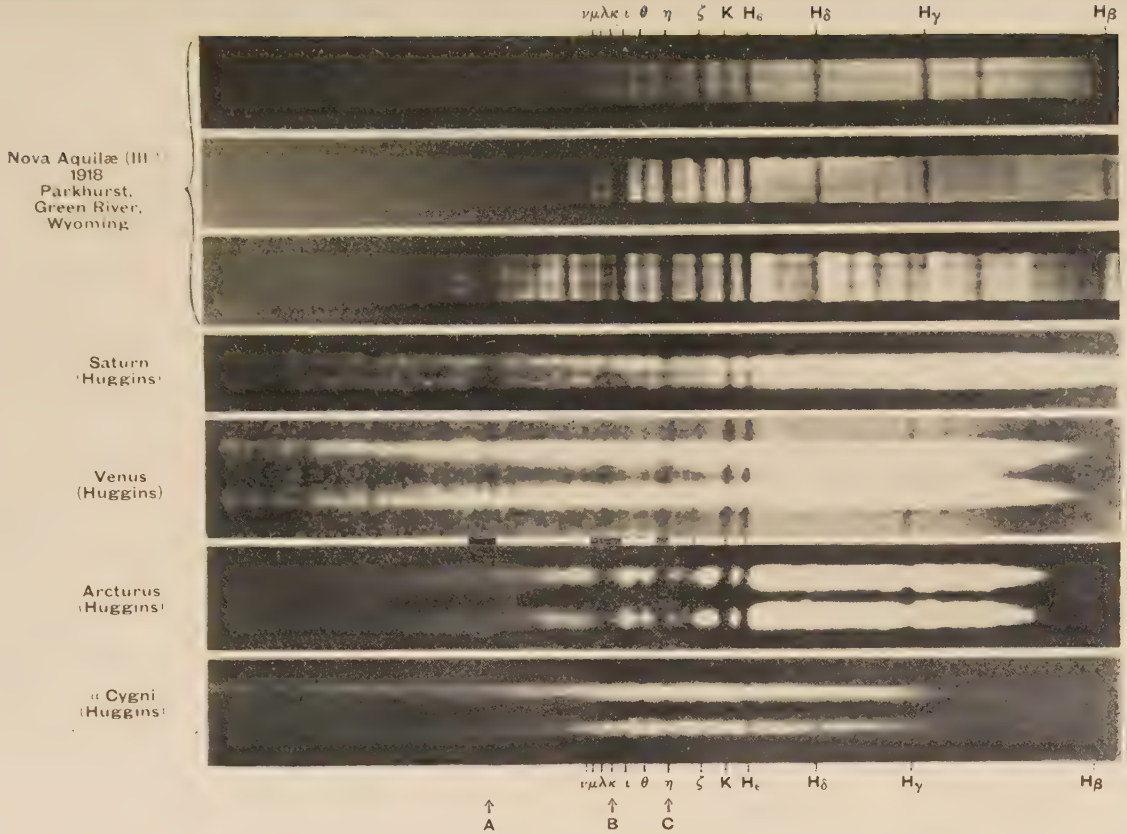
B = { λ3720–3750, covering the positions of hydrogen lines H_μ-H_λ-H_κ
λ3750–λ3800 ,, ,, ,, H_κ-H_ι

C = λ3830 – λ3840.

expressed in normal wave-lengths.

On June 10 all these groups are intensified, with some modifications of their relative structure.

In the comparison of the nova spectra with the solar and stellar spectra it must be noted that they were taken with prismatic spectrographs of different type, so that exact coincidences over a


νμλκ ι θ η ζ K H$_\epsilon$ H$_\delta$ H$_\gamma$ H$_\beta$
Nova Aquilæ (III) 1918 Parkhurst, Green River, Wyoming
1918 June 8^d 19^h 04^m
,, 9^d 20^h 02^m
,, 10^d 18^h 47^m
Saturn (Huggins)
1887 March 23
Venus (Huggins)
1879 June 17
Arcturus (Huggins)
1878 June 9
α Cygni (Huggins)
1878 July 29
νμλκ ι θ η ζ K H$_\epsilon$ H$_\delta$ H$_\gamma$ H$_\beta$
A
B
C

long range of spectrum cannot be expected. The scale was adjusted to give the best agreement from H and K to the ultra-violet, and the general agreement throughout this region appears sufficiently good to give confidence in the identifications. The determination of the origins of these groups is a matter of some difficulty, owing to the prevalence in this region of close groupings of lines of several elements, and the accompanying suggested origins are given provisionally.

Group A, λ3550–λ3585 (*estimated*).—The most satisfactory identification for this group is the prominent set of strong scandium spark lines ranging from λ3558 to λ3581.

Group B, λ3720–λ3750, λ3750–λ3800 (*estimated*).—This group shows considerable variation from June 9 to June 10, and it is to be expected that the changes may involve different substances on the two days. Considering the great intensity of the H and K lines, and also the presence of the strong titanium line at λ3685 (the strongest line of the titanium spark spectrum), it seems likely that the group may be due to the superposition of the strong titanium lines λ3759 and λ3761, together with λ3737, one of the strongest spark lines of calcium after H and K, over the hydrogen lines H_μ(3722·1), H_λ(3734·6), H_κ(3750·3), H_ι(3770·8).

Group C, λ3830–λ3840 (*estimated*).—The presence of the strong magnesium line λ4481 leads one to accept as the most probable origin for group C the strong triplet of magnesium λλ3830, 3832, 3838, superposed on the hydrogen line H_η(3835·6). Further evidence for the magnesium origin is also furnished by the abnormal intensity of the radiation group near λ5169 found on other photographs of the nova spectrum, probably due to the superposition of the green magnesium *b* group λλ5167·5, 5172·8, 5183·7 on the enhanced iron line λ5169·2.

[The line λ4481 has been ascribed to helium λ4471 by several observers at this stage of the nova spectrum, but as no modification of the helium spectrum is known which shows λ4471 without the strong line at λ4026, this origin appears doubtful at this stage. If helium were showing, there would also be expected a greater intensification of the H_ζ line by reason of the superposition of the strong helium line at λ3889. Adams and Joy give the line near λ4481 as due to magnesium in their spectrum of June 8 (*Ast. Soc. Pac.*, 1918 Aug., p. 251).]

However, an examination of these positions on Rowland's solar spectrum shows that the strongest lines near groups A and B are the prominent iron arc lines in that region, while group C may be accepted as mainly due to magnesium.

If then we have these peculiar groupings due to enhanced metallic lines in the nova, and to strong arc lines of another element in the Sun and Arcturus, the question arises as to the meaning of these apparent coincidences.

From other evidence we know that the characteristic lines of α Cygni are well represented in other parts of the spectrum of Nova Aquilæ on June 10, while it is seen that these peculiar ultra-violet

groupings are not prominent in the α Cygni spectrum. In discussing this question, Mr. Stratton makes the suggestion that as the first spectrum was of A type, it may be possible that we have here, in the ultra-violet, traces of an intermediate stage, approximating to G type, due to the superposition of arc absorption lines on the enhanced lines. No explanation is offered at present of this abnormality.

The Distance of the Milky Way. By Dr. A. Pannekoek.

(*Communicated by Dr. W. de Sitter.*)

1. By several modern researches formulæ have been deduced for the numbers of stars per square degree which depend on two variables, the limiting magnitude and the galactic latitude, thus: $N(m, l)$. These formulæ define our star system as a figure of revolution, in which the star density depends on distance and galactic latitude. In the direction of the galactic poles the density is least, and diminishes rapidly outwards; in the galactic plane the star density also decreases regularly with increasing distance, but at a slower rate. They represent our sun as lying in the midst of a flat star cluster whose densest parts measure some 1000 parsecs.

This character of our star system is not in accordance, however, with the appearance of the Galaxy. We see the Milky Way as a belt of luminous clouds, patches, and drifts, divided by less luminous regions or dark gaps and rifts. If we go in the direction of such a star-cloud, the star density, after we have left our central cluster, must increase at first on the nearer side of the centre of the cloud, and decrease on the farther side. The aspect of the Milky Way shows that by treating the galactic zone as a whole we intermingle parts of the universe of a great diversity of structure, and thus obscure the special character of this structure, viz. the aggregation of stars in clouds, separated by regions agreeing perhaps with the galactic poles. In studying the distribution of stars in our universe we must treat the different parts of the Galaxy, especially the great star-clouds and streams, separately, determine the function $N(m)$ for each, and derive from it the variation of star density with distance.

2. In a paper printed in the *Proceedings of the Amsterdam Academy*, 1910 June, a first attempt was made to find the function $N(m)$ for some parts of the Milky Way.* The parts chosen were the southern part of the great bright elliptic patch between β and γ Cygni, the poorer regions east of it, and a part of the Eastern stream between 10° and 20° declination, in Aquila and

* "Researches into the Structure of the Galaxy" (*Proc. Amst. Acad. Sci.*, 1910, p. 239).

Sagitta, including the bright patch of χ Aquilæ. The sources used for the numbers of stars of various magnitudes were :—

(1) The *Bonner Durchmusterung* for the magnitudes 6·5, 8, and 9 (limiting values of m for the three regions investigated 6·55, 6·56, and 6 56; 8·07, 8·08, and 8·08; 9·19, 9·22, and 9·24).

(2) The gauges of Sir William Herschel, which are especially numerous in these regions ($m = 13·90$).

(3) The star counts (as yet unpublished) made by Th. Epstein at Frankfort in the years 1877–1888 with a 6-inch telescope ($m = 12·51$).

(4) The *Carte du Ciel*, which furnishes two sorts of data, a brighter limit being provided by the Catalogue plates, a fainter one by the Chart plates. For the Cygnus regions we could only make use of the Potsdam Catalogue plates ($m = 11·73$); for the Aquila stream we had catalogue and chart plates of the Bordeaux zones $+14°$, $+16°$, $+17°$ ($m = 11·73$ and $13·20$).

The limiting magnitudes given in parentheses were found from the star numbers themselves. For a number of regions distributed over the whole sky, or at least over different galactic latitudes, the values of N were deduced, and with the aid of Kapteyn's table in *Groningen Public.*, **18**, which gives log N as a function of the limiting magnitude and the galactic latitude, a number of values for m were derived. For particulars of the values adopted, the former paper may be referred to. In *Groningen Public.*, **28**, Dr. Van Rhijn has since given corrections to reduce these magnitudes (based on the scale of Parkhurst) to the Harvard Scale. Using these corrections, the results of Table I. are found.

TABLE I.

	Cygnus Bright Patch.			Cygnus Faint Region.			Aquila Stream.		
	m.	log N.	$\frac{d \log N}{dm}$	m.	log N.	$\frac{d \log N}{dm}$	m.	log N.	$\frac{d \log N}{dm}$
B.D.	6·60	9·46		6·61	9·40		6·61	9·45	
,,	8·15	0·352	0·58	8·16	0 346	0·61	8·16	0·265	0·53
,,	9·29	1·031	0·60	9·32	1 020	0·58	9·34	0·898	0·54
C. d. C. Cat.	11·97	2·260	0·46	11·97	2·176	0·44	11·97	2·249	0·51
Epstein	12·89	2 629	0·40	12·89	2·513	0·37	12·89	2·557	0·33
C. d. C. Chart	...	...	0·62	...	...	0·38	13 76	3·205	0·74
Herschel	14·67	3·737		14·67	3·182		14·67	3·948	0·82

The values of the gradient $\frac{d \log N}{dm}$ show clearly the influence of a remote star-cloud; after the normal decrease, corresponding to the regular decrease of star density in the central cluster, the gradient increases for stars beyond the 13th magnitude. In the

Aquila stream this increase is more strongly marked than in the Cygnus patch. In the Cygnus region east of the bright patch, filled with faint light, the number of stars up to the 12th magnitude is nearly the same as in the bright patch itself; but the increase beyond the 13th magnitude is absent. The appearance of the Milky Way in this region also gives the impression of a bright star-cloud rising rather suddenly from the faint background on its eastern side.

3. To find theoretically the number of stars of magnitude m produced by such a star-cloud we assume the luminosity function as derived by Kapteyn,

$$\log \phi(i) = \text{const} - \frac{1}{\alpha^2}(M - M_0)^2.$$

For the distance r we introduce a new variable: the modulus of distance ρ, defined by

$$\rho = 5 \log r \quad (\rho = 0 \text{ for } \pi = 0''{\cdot}1).$$

If we suppose that in a star-cloud at distance r_0 the density decreases on both sides according to the law

$$\log \Delta = \text{const} - \frac{1}{\beta^2}(\rho - \rho_0)^2,$$

then the number of stars of magnitude m is given by

$$A_1(m) = \int_{-\infty}^{+\infty} \Delta(\rho) 10^{0{\cdot}6\rho - 1/\alpha^2(m - M_0 - \rho)^2} d\rho,$$

or

$$\log A_1(m) = \text{const} - \frac{1}{\alpha^2 + \beta^2}\{m - (\rho_0 + M_0 + 0{\cdot}3\beta^2)\}^2.$$

Now, if an agglomeration of stars appears to us as a cloud of 15° or 20° diameter, its extension along the line of sight being not greater, the constant β^2 must be < 1, whereas α^2 is nearly 30. Thus in this formula we may neglect wholly the extension of the star-cloud in depth, as its influence on the distribution of A_1 is insignificant compared with the dispersion caused by the luminosity curve. We will assume, therefore, an agglomeration of stars, lying all at the distance r. Then we have

$$\log A_1(m) = \text{const} - \frac{1}{\alpha^2}(m - m_0)^2,$$

where $m_0 = M_0 + \rho$.

To this A_1 must be added the number of stars, $A_0(m)$, filling the space between us and the cloud, which number is given by researches on the mean distribution in the regular star system. From the sum $A = A_0 + A_1$ the total number $N(m)$ of all stars brighter than m is computed, because the values of N, and not of A, are found directly in practical researches.

In our computations we have made use of the most recent researches on the luminosity curve by Kapteyn's method, made

by Dr. Schouten. His results for the number of stars $\phi(M)$ of absolute magnitude M * may be represented by the formula

$$\log \phi = \text{const} - 0{\cdot}029(M-9)^2.$$

This means that for $\pi = 0''{\cdot}01 (\rho = 5)$, $\pi = 0''{\cdot}001 (\rho = 10)$, $\pi = 0''{\cdot}0001 (\rho = 15)$, the most numerous stars appear as stars of a magnitude $m_0 = 14$, 19, and 24 respectively. Of course the formula has only a real meaning for the values of M between the limits −6 and +8, from which it has been deduced. Using these results we deduce

$$A_1(m) = C \cdot 10^{-0{\cdot}029(m-m_0)^2}.$$

For the numbers $A_0(m)$ we have taken the values found by Van Rhijn (and smoothed by Schouten) for the region of the galactic poles. In Table II. are given the results for log N (the values of A having been computed for integer values of m, the values of N correspond to the halves) and for the gradient $\frac{d \log N}{dm}$.

TABLE II.

m.	$\rho=15$, $m_0=24$. $C=2\times10^6$.		$\rho=16$, $m_0=25$. $C=5\times10^6$.		$\rho=17$, $m_0=26$. $C=12{\cdot}5\times10^6$.		$\rho=18$, $m_0=27$ $C=1/3\times10^8$.		$\rho=19$, $m_0=28$. $C=10^8$.	
	log N.	gr.	log N.	gr.	log N.	gr.	log N.	gr.	log N.	gr.
6	8·486	559	8·486	557	8·486	556	8·486	556	8·486	556
7	9·045	530	9·043	524	9·042	523	9·042	522	9·042	522
8	9·575	509	9·567	494	9·565	490	9·564	489	9·564	489
9	0·084	505	0·061	472	0·055	458	0·053	454	0·053	453
10	0·589	532	0·533	472	0·513	438	0·507	425	0·506	421
11	1·121	581	1·005	496	0·951	441	0·932	406	0·927	392
12	1·702	618	1·511	571	1·392	488	1·338	418	1·319	379
13	2·320	624	2·082	620	1·880	571	1·756	487	1·698	412
14	2·944	598	2·702	629	2·451	630	2·243	591	2·110	517
15	3·542	558	3·331	603	3·081	637	2·834	650	2·627	635
16	4·100	509	3·934	560	3·718	608	3·484	649	3·262	678
17	4·609	458	4·494	511	4·326	563	4·133	614	3·940	662
18	5·067	407	5·005	459	4·889	512	4·747	566	4·602	619
	5·474		5·464		5·401		5·313		5·221	
$A_0=A_1$	10·3		11·1		12·0		12·8		13·4	
Min.–Max.	8·7–12·7		9·6–13·7		10·6–14·7		11·4–15·6		12·2–16·4	
I	0·207		0·206		0·205		0·218		0·260	

For the bright stars the influence of the star-cloud is nearly (or wholly) zero, but for the higher magnitudes the number of cloud-stars A_1 increases much faster than A_0, and finally makes up the

* W. J. A. Schouten, *On the Determination of the Principal Laws of Statistical Astronomy*, p. 111.

overwhelming majority. The magnitude of transition, for which $A_1 = A_0$, is given at the foot of each table. All the values of log N and of their gradients show the same typical feature: after the normal decrease the gradient reaches a minimum, increases during an interval of about 4 magnitudes—just the interval in which the transition from minority to majority occurs—and then the gradient of log N, now due almost solely to cloud-stars, decreases again. The magnitudes for which the gradient reaches a minimum and a maximum are also given at the foot. They show that for each increase of ρ with unity the reversal of the gradient is displaced over nearly 1 mag. The place of this reversal, therefore, gives a means of finding the distance of the star-cloud.

In one case only the minimum, where the reversal begins, is indicated, as our data do not reach low enough to show the maximum. For the Cygnus patch the minimum lies at $m = 11{\cdot}5$, for the Aquila stream somewhat below the 12th magnitude. These values correspond respectively to $\rho = 18$ and 19. *So we find for the parallax of these parts of the Milky Way* 0″·000025 (*for the bright Cygnus patch*) *and* 0″·000016 (*for the Aquila stream*), *and for their distances* 40,000 *and* 60,000 *parsecs.*

In order to see how far these results depend on our suppositions the computations for $\rho = 18$ and 19 have been repeated, assuming as background A_0 the larger values of the star densities for $l = 20°$ at the border of the galactic zone, and in addition assuming for the density of the star-cloud and for the constant C values twice those first adopted. The results are contained in Table III.

TABLE III.

m.	$\rho=18$, $m_0=27$. A_0 for $l=20°$.		$\rho=19$, $m_0=28$. A_0 for $l=20°$.		$\rho=18$, $m_0=27$. $C=2/3\times10^8$.		$\rho=19$, $m_0=28$. $C=2\times10^8$.	
	9·766		9·766		9·564		9·564	
8	0·265	499	0·265	499	0·054	490	0·053	489
9	0·742	477	0·742	477	0·510	456	0·506	453
10	1·199	457	1·196	454	0·940	430	0·929	423
11	1·640	441	1·631	435	1·365	425	1·328	399
12	2·077	437	2·050	419	1·835	470	1·733	405
13	2·535	458	2·472	422	2·407	572	2·212	479
14	3·043	508	2·928	456	3·073	666	2·823	601
15	3·608	565	3·452	524	3·761	688	3·522	699
16	4·199	591	4·039	587	4·426	665	4·227	705
17	4·782	591	4·650	611	5·045	619	4·899	672
18	5·332	550	5·245	595	5·613	568	5·521	622
$A_0=A^1$	13·7		14·5		12·2		12·9	
Min.–Max.	11·8–16·5		12·5–17·2		10·7–14·8		11·7–15·7	

Comparing these results with Table II., we see that with a denser background the reversal of the gradient is weakened and displaced

towards the fainter magnitudes; by doubling the value of C the opposite effect is obtained, the reversal becoming stronger and steeper, and taking place at a lower value of m. In the first case the values of ρ corresponding to the minimum at 11·5 and 12·2 become half a unit smaller, in the second case somewhat less than a unit greater.

The first case is quite probable, as a star-cloud at such a distance is seen through a star-filled space much deeper than our universe reaches in the direction of the galactic poles. But we are not entitled to make arbitrary suppositions as to the density of the cloud or its constant C, for the total light of this cloud is observed as brightness of the galactic light. The brightness, *i.e.* the quantity of light per square degree, expressed by the number of stars of magnitude 1·0 is given by

$$I = \int C \,.\, 10^{-0\cdot029(m-m_0)^2-0\cdot4(m-1)} dm,$$

where the integral may be taken between the limits $\pm \infty$ because all the stars are telescopic. Its value is thus

$$I = C\sqrt{\frac{\pi \log e}{0\cdot029}}\, 10^{-0\cdot2(2m_0-8\cdot896)}.$$

This I is given at the foot of Table II. Now we do not know exactly the brightness of these patches; the measures of Yntema give 0·088 for the mean brightness of the Milky Way, and Van Rhijn gives 0·057 for $l = 10°$.* So it appears quite possible that a bright part of the Milky Way is 4 or 5 times brighter than this value, thus corresponding to the value of C adopted in Table II.; but it is doubtful whether we may double this value, as we do in the second half of Table III., by doubling the value of C.

4. For the Cygnus patch we have yet another means of finding its distance, by assuming that its light is simply superposed on the same background we are seeing as a faint nebulous light on its eastern side. Then the number of stars of each magnitude belonging to the star-cloud alone is found by subtracting the numbers belonging to the faint Cygnus region from the numbers for the bright patch. We thus have:

m.		N.	Log N.
9·32	11·07 − 10·48	= 0·59	9·77
11·97	182 − 150	= 32	1·50
12·89	426 − 326	= 100	2·00
14·67	5461 − 1522	= 3939	3·60

The number of stars brighter than m is proportional to the integral

$$\psi(x) = \int_{-\infty}^{x} 10^{-0\cdot029(m-m_0)^2} dm,$$

where $x = m - m_0$.

* *Groningen Public.*, 22, 34; 27, 55.

The values of $\log \psi$ and its gradients are:

$x = -18$	$\log \psi = 0{\cdot}110$	$\frac{d \log \psi}{dx} = 1{\cdot}049$	
-17	$1{\cdot}159$		$0{\cdot}992$
-16	$2{\cdot}151$		$\cdot 935$
-15	$3{\cdot}086$		$\cdot 878$
-14	$3{\cdot}964$		$\cdot 821$
-13	$4{\cdot}785$		

Between $12{\cdot}89$ and $14{\cdot}67$ the mean gradient is $0{\cdot}899$, corresponding to $x = -14{\cdot}9$; between $11{\cdot}97$ and $14{\cdot}67$ it is $0{\cdot}777$, corresponding to $x = -12{\cdot}7$. This gives for m_0 the values $13{\cdot}8 + 14{\cdot}9 = 28{\cdot}7$ and $13{\cdot}3 + 12{\cdot}7 = 26{\cdot}0$, corresponding to $\rho = 19{\cdot}7$ and $17{\cdot}0$. In this way we find for the distance values of the same order as above, the corresponding values of π being $0''{\cdot}000013$ and $0''{\cdot}000040$.

5. The foundation of these determinations of parallax and distance is the luminosity curve deduced by Kapteyn's method. As regards the numerical values deduced from observational data (reaching from $M = -6$ to $+8$) the curve deserves great confidence, and between these limits the formula adopted is fixed by them with great accuracy. Moreover, in computing the reversals of the gradient the formula has only been used as a formula of interpolation; we could have used the numerical values themselves as well, and so only the extreme values for $M = -5$ and -6, $m - m_0 = -14$ and -15, are somewhat uncertain. In computing the brightness, I, however, and also in the second method for the Cygnus patch, the formula is used beyond the limits of observational data. But practically the value of I is also nearly independent of the formula, as the few bright stars with $M < -6$, and the bulk of the very faint stars with $M > 8$, contribute only a small percentage to the value of the integral for I.

The weak point in our method of research lies in the lack of homogeneity in the observational data for N. The star counts of Herschel and Epstein are distributed fairly equally over the regions examined, but the *Carte du Ciel* plates are not, so irregularities in the distribution of the stars may more or less affect the course of the N. Our calculations must be considered as a first attempt to try a method which opens the possibility of studying the internal structure of the galactic system and the location and distribution of its streams and clouds in space as soon as the present scantiness of data is removed.

6. In Part 2 of his "Studies based on the Colours and Magnitudes in Stellar Clusters" Harlow Shapley estimated the diameter of our galactic system at 3000 parsecs, so that from the more remote star cluster Messier 13 it would appear like a nebulous patch of 5° diameter. This view may represent the actual ideas of the dimensions of the galactic system at that time. Afterwards, in his seventh paper, Shapley found that the system of the globular

clusters, which extends in the direction of galactic longitude 325 to a distance as far as 60,000 parsecs, must be connected with the galactic system, because nearer than 1000 parsecs from the galactic plane these clusters are absent. The results we have arrived at here are in accordance with these later views, as they place some bright parts of the Milky Way at a distance of 40–60,000 parsecs. So the starry masses of the Galaxy are spread over space as far as the remotest clusters, and clearly both belong together to one system. In this system the dense agglomerations of stars are spread over a flat disc of about 2000 parsecs thickness, and in the empty space above and below it the globular clusters are dispersed. Shapley explained the absence of these clusters from the galactic disc by the absorption of the dark nebulous masses filling this space. It may perhaps also be explained by assuming that globular clusters, when coming from the void space into the star-filled galactic regions, are gradually broken up and dispersed into open clusters by the attraction of these stars.

Now, Shapley's result, that in the universe of globular clusters the sun occupies a very eccentric position, is contrary to the common view, which places the sun in our galactic system not far from the centre. Therefore, it must be emphasised that Shapley's result is wholly in accordance with the aspect of the Milky Way. For the hemisphere with centre at $\lambda = 325^\circ$, where judging by the abundance of globular clusters our universe reaches farthest, we observe in the direction of Sagittarius a series of round or irregular and rather small bright patches and clouds, which probably lie at a great distance. At the opposite side of the heavens we have at $\lambda = 100^\circ$ to 140° the faint region of Perseus, where the galactic light nearly vanishes; on both sides of it the light band gradually becomes brighter. So it is very probable that the galactic star masses on the side of Scorpio and Sagittarius reach many times farther than on the opposite side. The sun must then be situated near to the limit of the system in the direction of Perseus.

Note on the Blue-Violet Absorption of Venus.

Arthur W. Clayden, M.A.

In Mr. Evershed's interesting report on the work of the Kodaikánal and Madras Observatories, published in the February number of the *Monthly Notices*, a passage occurs which suggests some comment. It runs thus:—

"In photographing the spectrum of Venus with the grating spectrograph in the blue and violet regions, it was noticed that longer exposures were required than is necessary when the image of a brightly-illuminated cloud is brought on to the slit. Direct comparisons of the spectra in a low-dispersion prism spectrograph,

using a parabolic mirror to form the image of Venus, showed that, with exposures regulated to give equal density in the green region, the Venus spectra are much weaker in the violet than the cloud spectra, suggesting that the atmosphere of Venus is devoid of clouds, or, if these are present, the atmosphere above them must be strongly absorptive for the violet rays."

Surely the conclusion suggested is that the atmosphere of Venus is like our own, if not chemically, at least physically. The violet and blue rays are just those which are absorbed or scattered in the upper portions of our own atmosphere, and to such an extent that it has often been said that if we could see the sun outside this absorptive envelope it would look blue to our eyes.

The spectrum of a brightly-lighted cloud is formed by light which has passed once only through our atmosphere.

The spectrum of Venus is formed by light which has passed similarly once through our atmosphere and *twice through* that part of the atmosphere of the planet which is above the reflecting surface.

If the planet's atmosphere is similar to ours, and the reflecting surface is white cloud similar to ours, we ought therefore to expect something like twice or three times the same kind of absorption.

Moreover, it is believed that the particles which account for much of the scattering of short wave-lengths in our atmosphere are due to ions formed by the action of ultra-violet light. The intensity of solar radiation per unit of area on Venus is nearly twice as great as it is upon the earth. Hence we might expect a greater number of the necessary particles, and therefore a greater proportionate loss of blue and violet rays in the case of Venus.

Again, the proportion of absorption depends on the thickness of atmosphere traversed, increasing rapidly with large angles of incidence. The light from a bright cloud would have traversed our atmosphere at no very large angle from the vertical, whereas the light reflected from Venus must all have traversed its upper atmosphere obliquely in one or both of its journeys, and some must have passed almost tangentially.

Hence, if the two atmospheres are similar, we should expect greater absorption in the violet and blue.

In fact the observations do not at all suggest absence of clouds, but rather that the shining surface of the planet is made of clouds, and that they are seen through an upper atmosphere like our own in its selective absorptive power and of approximately equal density.

Micrometrical Measures of Double Stars in 1918.

By Rev. T. E. R. Phillips, M.A.

The stars in the following lists were measured in 1918 with the 8-inch refractor belonging to the Society :—

Star.	1920. R.A.	1920. Dec.	Mags.	Position-Angle.	Distance.	No. of Nights.	Date.
	h m	° ′		°	″		
Σ 60 (η Cassiop.)	0 44·0	+57 24	4·0, 7·6	256·0	6·92	4	1918·637
73 (36 Androm.)	0 50·7	23 12	6·2, 6·8	49·0	0·90	4	·787
138	1 31·8	7 16	7·3, 7·3	43·3	1·79	3	·926
178	1 47·8	10 25	7·8, 7·8	199·3	3·24	3	·937
180 (γ Arietis)	1 49·1	18 54	4·2, 4·4	359·4	8·13	3	·862
186	1 51·7	1 27	7·2, 7·2	43·3	1·10	3	·942
OΣ 38 (γ_2 Androm.)	1 59·0	41 57	5·0, 6·2	112·6	0·52	4	·836
Σ 425	3 35·1	33 52	7·3, 7·3	86·9	2·44	2	·913
535	4 18·8	11 12	6·7, 8·2	324·1	1·24	2	·922
OΣ 95	5 0·8	19 42	6·5, 7·0	320·0	0·73	2	·977
A. G. Clark I. (Sirius)	6 41·6	−16 35	1·4, 10·0	72·1	10·87	6	·162
				68·9	10·88	2	·799
Σ 982 (38 Geminorum)	6 50·1	+13 17	5·4, 7·7	156·5	6·59	2	·987
1110 (Castor)	7 29·5	32 4	2·7, 3·7	215·9	4·78	4	·916
1126	7 35·8	5 24	7·2, 7·5	148·3	1·26	3	·242
1130	7 37·3	9 54	8·4, 8·9	171·5	1·00	3	·243
1196 (ζ Cancri) AB	8 7·6	17 54	5·5, 6·2	291·3	0·96	3	·188
BC	...	...	...	110·7	4·87	2	·186
1356 (ω Leonis)	9 24·2	9 25	6·2, 7·0	128·8	0·98	3	·291
1424 (γ Leonis)	10 15·5	20 15	2·0, 3·5	116·1	3·63	4	·211
1536 (ι Leonis)	11 19·7	11 1	3·9, 7·1	39·9	1·86	3	·344
1670 (γ Virginis)	12 37·7	− 1 0	3·6, 3·6	323·2	5·78	3	·345
1865 (ζ Böotis)	14 37·3	+14 4	3·5, 3·9	138·5	0·81	6	·302
1877 (ε Böotis)	14 41·5	27 21	3·0, 6·3	333·1	2·58	3	·361
1888 (ξ Böotis)	14 47·7	19 26	4·7, 6·6	78·0	2·24	5	·154
				77·0	2·23	5	·363
				76·2	2·32	5	·424
				76·9	2·30	5	·528
		Mean of all measures :—		77·0	2·75	20	·370
1932 (1 Coronæ)	15 14·9	27 7	5·6, 6·1	11·1	0·65	3	·357
1937 (η Coronæ)	15 19·9	30 34	5·5, 6·0	73·8	0·75	3	·358

Star.	1920. R.A.	1920. Dec.	Mags.	Position-Angle.	Distance.	No. of Nights.	Date.
	h m	° ′		°	″		
Σ 2032 (σ Coronæ)	16 11·7	34 1	5·0, 6·1	217·9	4·76	3	1918·405
2055 (λ Ophiuchi)	16 26·9	2 10	4·0, 6·1	76·5	0·90	3	·384
2140 (α Herculis)	17 11·0	14 29	3·0, 6·1	110·7	4·57	3	·419
2244	17 53·0	0 4	6·9, 7·1	276·2	0·78	3	·394
2245	17 52·9	18 20	7·0, 7·0	292·5	2·54	3	·415
2264 (95 Herculis)	18 58·4	21 36	4·9, 4·9	258·6	6·02	3	·457
2272 (70 Ophiuchi)	18 1·4	2 31	4·1, 6·1	135·9	5·09	5	·363
				135·8	5·12	5	·422
				134·9	4·99	5	·554
				134·2	5·01	5	·666
	Mean of all measures:—			135·2	5·05	20	·501
2375	18 41·5	5 24	6·2, 6·6	113·9	2·26	3	·477
2382 (ϵ_1 Lyræ)	18 41·7	39 35	4·6, 6·3	8·5	2·90	3	·392
2383 (ϵ_2 Lyræ)			4·9, 5·2	119·8	2·33	3	·392
2384	18 38·5	67 4	8·0, 8·5	307·4	0·76	2	·398
2579 (δ Cygni)	19 42·5	44 55	3·0, 7·9	280·5	1·69	3	·406
2583 (π Aquilæ)	19 44·9	10 25	6·0, 6·8	113·6	1·54	3	·536
OΣ 413 (λ Cygni)	20 44·3	36 12	5·0, 6·3	49·7	0·69	3	·415
Σ 2822 (μ Cygni)	21 40·5	28 23	4·0, 5·0	134·6	1·47	2	·590
3041 BC	23 43·8	16 38	7·3, 8·1	179·6	3·60	3	·766
3050	23 55·4	33 17	6·0, 6·0	224 9	2·24	3	·893

Notes.

A. G. Clark I. (Sirius). The comes is at present quite an easy object with 8 inches in good atmospheric conditions.

Σ 1110 (Castor). The measures of the last few years have exhibited a very decided diminution in the distance, while the angle is diminishing very slowly.

Σ 1888 and Σ 2272. Much attention has again been given to ξ Böotis and 70 Ophiuchi, but there is nothing calling for remark in the measures.

Headley Rectory, Epsom.
1919 *May* 5.

Note on Espin's List of New Double Stars. By Eric Doolittle.

This interesting list of the most recent discoveries, which appears in the *Monthly Notices*, 79, 212–217, comprises 71 pairs found by Espin and 79 pairs discovered by Milburn—150 pairs altogether. The following errata and identities may be noted:—

Espin 1741. The Decl. is 4′ too large.

1758. This triple was first measured by Fox (*Annals, Dearborn Obs.*, i.).

1911·73 179°·6 5″·42 10·0 11·7 *AB* 3*n*.

1911·75 347 ·5 29 ·03 10·5 *AC* 2*n*.

Milburn 47. The number of nights is omitted in *AC*.

55. This is Roe 76. The position from measures from DM 40° (783), 8·8 magnitude, is (1880·0), R.A. = 3 hrs. 26 min. 0·8 secs.; Decl. = 40° 23′·5. The only prior measure is by Roe:—

1914·11 30°·4 4″·18 9·5 9·8 3*n* (*A.N.*, 4762).

64. This was discovered by Pettit in 1916 (= Pettit 11). His measure is:—

1916·85 6°·6 4″·24 9·1 10.4 3*n* (*A.J.*, 715).

68. Identical with Jonckheere 1023, but the identification of this was erroneous in *J.A.*, ii. p. 16. That given by Milburn is correct. The pair is also A.G. Lund, 5451, 9·5 magnitude. The earliest measure is:—

1911·75 171°·6 4″·20 9·3 9·5 Jonckheere 2*n*.

96. Identical with Doolittle 18 (*General Catalogue*, 11780), but the identification in the *G.C.* is in error. That given here is correct. The only prior measure is:—

1899·11 44°·8 2″·70 9·2 10·5 Doolittle 4*n* − 3*n*.

103. The angles by Milburn and Espin differ by 16°·2. Misprint in one of the measures (?).

The Flower Observatory:
1919 *March* 28.

A New Variable Star in Ursa Major. By Rev. T. E. Espin.

In the course of the zone observations with the 17¼-in. reflector, Mr. W. Milburn has been giving particular attention to the red stars. The star B.D. + 59°·1401, which in the B.D. is entered as of mag. 9·3, I observed 1894 March 24 as 8·6 mag. Mr. Milburn has made the following recent observations of it:—

1919 March	5,	9·0.
April	9,	9·0.
April	12,	9·0.

On April 18 I made it 8·8 with the 24 in.

A series of four negatives made by Mr. Lawrence Richardson, covering this region, were examined by Mr. Milburn, also one by Mr. Offord, and plates 67 and 68 of the Harvard Map of the Sky, and show conclusively that the star is variable. The star B.D. + 59°·1398 was compared with certain stars whose photometric magnitudes are given in *H.A.*, **70**, and the magnitudes estimated at 9·7, while a star not in the B.D., but which by rough measures on the photographs is 15 sec. *p*, and 4′ *s* of the variable, was in the same way estimated as 10·7. Mr. Milburn then compared the variable with these stars, with the following results:—

1901 April	22,	10·0.	Offord's photo.
1902 May	1,	10·5.	Harvard Map of the Sky.
1905 Feb.	26,	9·9.	„ „ „
1916 March	31,	10·4.	Richardson's negative.
1916 April	4,	10·0.	„ „
1917 March	17,	10·6.	„ „
1917 April	17,	10·8.	„ „

The results show quite conclusively that the star is variable.

The spectrum was observed by me as Bands? III.? 1894 March 24, *A.N.*, 3286, where it is numbered 988.

The place is:—$11^h\ 39^m\ 13^s{\cdot}6 + 59^\circ\ 2'{\cdot}3$, 1855.

$11^h\ 41^m\ 40^s + 58^\circ\ 47'{\cdot}6$, 1900 (Krüger, *Neuer Kat. Farb. Sterne*, No. 2656).

Observations of (*Ch.* 678) *U Persei in* 1911–19.

By A. N. Brown, M.A.

The 313 observations given below are in continuation of those made by me in 1909–11, and published in *Monthly Notices*, 71, 520–23. Light curves have been drawn from them, from which the following tables have been compiled. They include the two maxima and one minimum observed in 1909–11 :—

	Maxima.			Minima.		
No.	Date.	Julian Date.	Mag.	Date.	Julian Date.	Mag.
(1)	1909 Nov. 16	8627	8·1	1910 May 11	8803	10·4
(2)	1910 Oct. 14	8959	8·3	1911 Mar. 28	9124	10·4
(3)	1911 Sept. 12±	9292	8·2	1912 Feb. 17	9450	10·4
(4)	1912 June 17	9571	8·2	1913 Jan. 2	9770	10·4
(5)	1913 Apr. 16	9874	8·1	1913 Nov. 22	0094	10·6
(6)	1914 Mar. 2	0194	7·8	1914 Oct. 2	0408	10·5
(7)	1915 Jan. 25	0523	8·0	1915 July 24	0703	10·5
(8)	1916 Jan. 29	0892	8·1	1916 June 26	1041	10·5
(9)	1916 Oct. 19	1156	7·9	1917 Apr. 16	1335	10·5
(10)	1917 Sept. 16	1488	8·2	1918 Mar. 17	1670	10·6
(11)	1918 July 26	1801	8·1	1919 Feb. 28	2018	10·4

The symbol ± above implies that the date is somewhat uncertain owing to paucity of observations. From the above it will be seen that the star varied from 7·8–8 3 magnitude to 10·4–10·6 magnitude (Hagen's scale).

It is noticeable that, as shown in the following table, there are very considerable departures from the mean period (derived from maxima and minima), which is 319·5 days :—

No.	(*a*).	(*b*).	(*c*).	(*d*).	No.	(*a*).	(*b*).	(*c*).	(*d*).
(1)	...	...	...	176	(7)	329	295	115	180
(2)	332	321	156	165	(8)	369	338	189	149
(3)	333±	326	168	158±	(9)	264	294	115	179
(4)	279±	320	121	199	(10)	332	335	153	182
(5)	393	324	104	220	(11)	313	348	131	217
(6)	320	314	100	214					

In the above table all the intervals are given in days, and the four columns show as follows :—(*a*) the interval between successive maxima; (*b*) the interval between successive minima; (*c*) the time taken for the rise from minimum to maximum; (*d*) the time taken for the fall from maximum to minimum.

Further points which strike me, as an observer, are :—

1. That, while the rise is usually more rapid than the fall, this is not always the case. Thus the rise numbered 8 above occupied 189 days. The succeeding fall is completed in 149 days.

2. That after No. 9 maximum above there are indications of a secondary maximum of 8·1 magnitude having occurred on 1916 December 27, followed by an immediate fall to minimum.

3. The light curve shows marked changes at different times. Sometimes the increase or decrease of the light is steady, sometimes fitful, and interrupted by marked halts or pauses, the existence of which seems well established by the observations.

4. The variable is usually at or near its maximum light for a much longer period than its minimum light, and at each phase the curve is invariably flat.

Of the four columns in which the observations are arranged, I think that the third alone needs any explanation. The majority of the comparisons were made by the "fractional" method, but each is here expressed in steps, the value of a step in every case being ·1 magnitude. Sometimes, however, the estimates were made in steps, and in order to distinguish the two methods a semicolon has always been placed between each separate estimate. Thus on 1911 October 31 there are two fractional estimates; on 1912 January 28 a fractional followed by a step estimate; on 1916 April 14 three step estimates follow each other. In deducing the magnitude equal weight has been given to fractional and step estimates. The comparison stars used, a list of which is given after the observations, are all numbered as in the Catalogue of Hagen's *Atlas Stell. Var.*, and Hagen's magnitudes have been adopted throughout. The negative sign after a comparison star implies that the variable was fainter, the positive that it was brighter, than the star in question.

The telescopes used were 5-inch and 3-inch refractors, with powers varying from 25 to 120.

The Observations.

Date. 1911		Julian Date.	Comparisons.	Ded. Mag.	Date. 1911		Julian Date.	Comparisons.	Ded. Mag.
Jan.	30	9067	11+1	9·0	Apr.	23	9150	33−2, 37+2	10·3
	31	9068	7−4, 11+1	9·0	May	4	9161	18−1, =21	9·7
Feb.	19	9087	22−1·5, 33+3	9·8		10	9167	18−1, 19+0·5	9·6
	20	9088	21−3, 33+1	10·0		24	9181	11−1, 16+2	9·2
	22	9090	23−3, 33+1	10·0		29	9186	11−1, 16+2	9·2
	25	9093	22−2·5, 33+1	10·0	June	9	9197	7−4, 12+1	9·1
Mar.	2	9098	22−2·5, 33+1	10·0		27	9215	=7	8·6
	7	9103	33−3, 37+1	10·4	July	9	9227	4−3·5, =7	8·6
	20	9116	33−3, 37+1	10·4		21	9239	4−3·5, 7+1	8·5
	21	9117	33−3, 37+1	10·4		25	9243	4−3·5, 7+1	8·5
	25	9121	33−3, 37+1	10·4		31	9249	4−3, 7+1	8·5
Apr.	3	9130	33−3, 37+1	10·4	Oct.	1	9311	=4	8·2
	15	9142	33−3, 37+1	10·4		18	9328	4−3·5, 7+1	8·5
	19	9146	33−2·5, 37+1	10·4		25	9335	4−3·5, 7+1	8·5

The Observations—continued.

Date.		Julian Date.	Comparisons.	Ded. Mag.
1911				
Oct.	28	9338	4 − 3·5, 7 + 0·5	8·6
	31	9341	6 − 1, 8 + 0·5; 7 − 1, 11 + 3·5	8·7
Nov.	12	9353	7 − 3, 11 + 2	8·9
	15	9356	7 − 4, 11 + 1	9·0
	29	9370	7 − 4, 11 + 0·5	9·0
Dec.	9	9380	7 − 4, 11 + 0·5	9·0
	19	9390	16 − 0·5, 18 + 1	9·4
1912				
Jan.	26	9428	33 − 3, 37 + 1	10·4
	28	9430	33 − 3, 37 + 1; 39 + 2	10·3
Feb.	11	9444	33 − 3, 37 + 1	10·4
Mar.	6	9468	33 − 3, 37 + 1	10·4
	7	9469	33 − 2, 37 + 2	10·3
	10	9472	33 − 3, 37 + 1	10·4
Apr.	3	9496	19 + 1	9·5
May	17	9540	4 − 3, 7 + 1	8·5
June	10	9564	= 4	8·2
	20	9574	= 4	8·2
July	13	9597	4 − 3, 7 + 0·5	8·5
	15	9599	4 − 1, 7 + 3·5	8·3
Sept.	2	9648	7 − 1, 11 + 4	8·7
	17	9663	7 − 4, 11 + 1	9·0
	20	9666	7 − 4, 11 + 1	9·0
	22	9668	7 − 4, 11 + 1	9·0
Oct.	2	9678	7 − 4, 11 + 0·5	9·0
	4	9680	12 − 1, 16 + 1·5	9·3
	8	9684	12 − 1·5, 16 + 1	9·3
	9	9685	12 − 1·5, 16 + 1	9·3
	10	9686	16 + 1·5	9·3
	14	9690	12 − 1, 16 + 1·5	9·3
Nov.	27	9734	22 − 2, 33 + 2	9·9
	30	9737	33 − 3, 37 + 1·5	10·4
Dec.	10	9747	33 − 3, 37 + 1·5	10·4
	12	9749	33 − 3, 37 + 1	10·4
	16	9753	33 − 3, 37 + 1	10·4
Dec.	18	9755	33 − 3, 37 + 1	10·4
	29	9766	33 − 3, 37 + 1	10·4
1913				
Jan.	1	9769	33 − 3, 37 + 1	10·4
	5	9773	33 − 3, 37 + 1	10·4
	12	9780	33 − 3, 37 + 1	10·4
	26	9794	33 − 3, 37 + 1·5	10·4
	31	9799	33 − 3, 37 + 1	10·4
Feb.	8	9807	33 − 2·5, 37 + 2	10·3
	22	9821	= 11	9·1
Mar.	2	9829	7 − 3, 11 + 1·5	8·9
	8	9835	4 − 3·5, 7 + 1	8·5
	11	9838	7 − 4, 11 + 1	9·0
	23	9850	4 − 2·5, 7 + 2	8·4
	28	9855	3 − 1·5, 4 + 1	8·1
May	1	9889	4 − 1, 7 + 3	8·3
	6	9894	3 − 1·5, 4 + 1	8·1
	30	9918	4 − 3·5, = 7	8·6
June	4	9923	4 − 3, 7 + 1	8·5
	27	9946	4 − 3, 7 + 1	8·5
July	3	9952	4 − 3, 7 + 0·5	8·5
	11	9960	7 − 1, 11 + 4	8·7
	20	9969	7 − 4, 11 + 1	9·0
	24	9973	7 − 3, 11 + 1·5	8·9
	29	9978	7 − 4, 11 + 1	9·0
Aug.	29	0009	= 16	9·4
Sept.	6	0017	16 − 0·5	9·5
	8	0019	16 + 1	9·3
	24	0035	18 − 1, 19 + 0·5	9·6
	28	0039	19 − 1, 21 + 0·5	9·7
Nov.	1	0073	37 − 1	10·6
Dec.	14	0116	33 − 2, 37 + 2	10·3
	18	0120	33 − 3, 37 + 1	10·4
	28	0130	19 − 1, = 21; 22 + 1	9·6
	30	0132	16 − 1, = 18	9·5

The Observations—continued.

Date.		Julian Date.	Comparisons.	Ded. Mag.
1914				
Jan.	3	0136	7 − 4, 11 + 0·5	9·0
	5	0138	7 − 5, 12 + 0·5	9·1
	23	0156	4 − 3, 7 + 0·5	8·5
	26	0159	4 − 3, 7 + 1	8·5
Feb.	2	0166	4 − 2·5, 7 + 1	8·5
	15	0179	2 − 9, 3 + 1	7·9
	18	0182	2 − 8·5, 3 + 2	7·8
	22	0186	2 − 7, 3 + 2	7·8
	24	0188	2 − 7, 3 + 2	7·8
Mar.	16	0208	3 + 2	7·8
	21	0213	3 + 1	7·9
	27	0219	3 + 1	7·9
	31	0223	3 + 2	7·8
Apr.	25	0248	3 − 4, 7 + 2	8·4
	28	0251	3 − 2, 7 + 4	8·2
May	2	0255	3 − 5, 7 + 1	8·5
	16	0269	3 − 5, 7 + 1	8·5
	21	0274	3 − 5·5, 7 + 1	8·5
	25	0278	3 − 5·5, 7 + 1	8·5
June	4	0288	7 − 1·5, 11 + 4	8·7
	14	0298	7 − 1, 11 + 4	8·7
	29	0313	7 − 2, 12 + 4	8·8
July	4	0318	7 − 5, 12 + 1	9·1
	9	0323	12 + 1	9·1
	16	0330	= 12	9·2
	29	0343	16 − 0·5, 18 + 1	9·4
Aug.	14	0359	19 − 0·5, 22 + 1	9·6
	26	0371	19 − 0·5, = 21	9·6
	29	0374	= 22	9·7
Sept.	5	0381	33 − 2·5, 37 + 2	10·3
	11	0387	33 − 2·5, 37 + 1	10·4
	17	0393	33 − 3, 37 + 1	10·4
	19	0395	= 37	10·5
	24	0400	= 37	10·5
	27	0403	= 40	10·6
	29	0405	= 37	10·5
Oct.	11	0417	= 37	10·5
	18	0424	22 − 3, 33 + 1	10·0
Nov.	1	0438	7 − 4, 11 + 1	9·0
	14	0451	7 − 3, 11 + 2	8·9
	18	0455	7 − 1·5, 11 + 4	8·7
Dec.	1	0468	4 − 3, 7 + 1	8·5
	16	0483	3 − 5, 7 + 1	8·5
1915				
Jan.	5	0503	= 3	8·0
	19	0517	= 3	8·0
Feb.	15	0544	= 3	8·0
	28	0557	3 + 3	7·7
Mar.	13	0570	3 − 4, 7 + 1	8·5
Apr.	17	0605	3 − 4, 7 + 1	8·5
	23	0611	3 − 5, 7 + 1	8·5
	29	0617	= 7	8·6
May	9	0627	7 − 2, 12 + 4	8·8
	23	0641	7 − 5, 12 + 1	9·1
	30	0648	12 − 1, 16 + 1	9·3
June	7	0656	18 − 0·5, = 19	9·6
July	11	0690	33 − 3, 37 + 1	10·4
	15	0694	33 − 3, 37 + 1	10·4
	28	0707	= 37	10·5
Sept.	11	0752	18 − 0·5, 19 + 0·5	9·6
	19	0760	16 − 0·5, 18 + 0·5	9·5
	25	0766	7 − 5, 12 + 0·5	9·1
	29	0770	7 − 4, 11 + 1	9·0
Oct.	4	0775	7 − 4, 11 + 1	9·0
	18	0789	7 + 1	8·5
	29	0800	4 − 3, 7 + 1	8·5
Nov.	5	0807	4 − 3, 7 + 1	8·5
	14	0816	7 + 1	8·5
	27	0829	3 − 5, 7 + 1	8·5
Dec.	8	0840	3 − 5, 7 + 1	8·5
1916				
Jan.	7	0870	3 − 1, 7 + 4	8·2
	13	0876	3 − 1, 7 + 5	8·1
	15	0878	3 − 1	8·1

The Observations—continued.

Date.		Julian Date.	Comparisons.	Ded. Mag.
1916				
Jan.	22	0885	3 − 1	8·1
Feb.	5	0899	3 − 1	8·1
	21	0915	3 − 1	8·1
Apr.	2	0956	7 − 4, 12 + 1·5	9·0
	5	0959	7 − 4, 11 + 1	9·0
	9	0963	7 − 3·5, 11 + 1	9·0
	14	0968	7 − 4 ; 11 + 1 ; 12 + 1·5	9·0
May	8	0992	21 + 1 ; 22 + 1	9·6
	18	1002	19 − 1, = 21	9·7
	28	1012	= 33	10·1
June	7	1022	37 + 1	10·4
	21	1036	= 37 ; = 39	10·5
	30	1045	= 37 ; = 39	10·5
July	3	1048	= 37 ; = 39	10·5
	8	1053	= 37 ; = 39	10·5
	22	1067	19 + 0·5 ; 21 + 1 ; 22 + 1	9·6
	29	1074	12 − 1, 16 + 1	9·3
Aug.	4	1080	7 − 3·5, 11 + 1	9·0
	19	1095	= 6 ; 7 + 1	8·5
	30	1106	3 − 5, 7 + 1	8·5
Sept.	4	1111	3 − 4, 7 + 1	8·5
	6	1113	3 − 4, 7 + 1	8·5
	13	1120	3 − 4, 7 + 1	8·5
	19	1126	3 − 3, 7 + 3	8·3
	21	1128	3 − 1, 7 + 4	8·2
	25	1132	3 + 1	7·9
	27	1134	3 + 2	7·8
Oct.	15	1152	3 + 1	7·9
	19	1156	3 + 1	7·9
	20	1157	3 + 1	7·9
	25	1162	3 + 1	7·9
	30	1167	3 + 1	7·9
	31	1168	3 + 1	7·9
Nov.	12	1180	3 + 1	7·9
	20	1188	3 − 1	8·1
	22	1190	3 − 1, 7 + 4	8·2
Nov.	26	1194	3 − 2	8·2
Dec.	4	1202	3 − 4, 7 + 2	8·4
	25	1223	3 − 2, 7 + 4	8·2
	27	1225	3 − 1, 7 + 4	8·2
1917				
Jan.	4	1233	3 − 3·5, 7 + 2	8·4
	27	1256	7 − 1	8·7
Feb.	6	1266	7 − 4, 12 + 2	9·0
	9	1269	7 − 4, 12 + 1·5 ; 8 − 3	9·0
	15	1275	7 − 5, 12 + 1	9·1
	26	1286	12 − 1·5, 16 + 1	9·3
Mar.	14	1302	= 22	9·7
	19	1307	23 − 1 ; 24 − 1 ; 33 + 1	9·9
	22	1310	22 − 2, 33 + 2	9·9
	27	1315	19 − 0·5, = 21	9·7
Apr.	20	1339	= 37 ; = 39	10·5
	23	1342	33 − 3, 37 + 1 ; 39 + 1	10·4
	25	1344	33 − 2 ; 37 + 1 ; 39 + 1	10·4
May	4	1353	= 22	9·7
	8	1357	33 − 1·5, 37 + 3	10·2
	28	1377	19 − 3, 33 + 1·5	9·9
June	4	1384	19 − 0·5, = 22 ; 21 + 0·5	9·7
	12	1392	16 + 1 ; 18 + 2	9·3
	17	1397	16 + 2 ; 18 + 3	9·2
	21	1401	7 − 6·5, 16 + 1	9·3
	26	1406	7 − 4, 11 + 1 ; 16 + 3	9·0
July	1	1411	7 − 6, 16 + 1·5	9·2
	15	1425	7 − 1, 11 + 4	8·7
	19	1429	7 − 2, 13 + 4 ; = 8	8·8
	22	1432	7 − 1	8·7
	25	1435	3 − 5, 7 + 1	8·5

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1917			
Aug. 11	1452	3−4, 7+2	8·4
15	1456	3−4, 7+2	8·4
18	1459	3−4, 7+2	8·4
22	1463	3−4·5, 7+2	8·4
Sept. 1	1473	3−4, 7+2	8·4
10	1482	3−3, 7+3	8·3
12	1484	3−2, 7+4	8·2
14	1486	3−2, 7+4	8·2
20	1492	3−2, 7+4	8·2
24	1496	3−3, 7+3	8·3
Oct. 4	1506	3−4, 7+2	8·4
23	1525	3−4, 7+1	8·5
Nov. 4	1537	3−4, 7+1	8·5
7	1540	3−5, 7+1	8·5
Dec. 2	1565	7−1, 13+5	8·7
6	1569	7−3; 11+3; 13+3	8 9
14	1577	7−5, 13+1	9·1
1918			
Jan. 3	1597	16+0·5	9·4
7	1601	16−1; =17; =18	9·5
11	1605	18−1; =19; 22+2; 23+2	9·6
20	1614	=22	9·7
Feb. 8	1633	37−0·5	10·6
16	1641	37−1; =39	10·6
24	1649	37−1, 42+2; 39−1	10·6
Mar. 1	1654	37−0·5; 39−1	10·6
7	1660	=37; 39−1; 42+2	10·6
9	1662	37−1; 39−1; 42+2	10·6
16	1669	37−1; 39−1; 42+2	10·6
24	1677	37−2; 39−2; =42	10·7

Date.	Julian Date.	Comparisons.	Ded. Mag.
1918			
Mar. 30	1683	37−1; 39−1; 42+2	10·6
Apr. 6	1690	39−1, 42+2	10·6
12	1696	37−1; 39−1	10·6
14	1698	=37; =39; 42+2	10·5
27	1711	=33; 34+1	10·2
May 2	1716	28−1; 33+1	10·0
10	1724	=16; 17+1; 24+2	9·4
13	1727	13−1, 17+2; 16+1	9·3
June 5	1750	7−1, 13+5	8·7
13	1758	=6; 7+1	8·5
22	1767	3−5, 7+1	8·5
29	1774	3−2, 7+4	8·2
July 2	1777	3−1; 7+4	8·2
8	1783	3−1, 7+5	8·1
12	1787	3−1; 7+3	8·2
18	1793	3−1, 7+5	8·1
29	1804	3−1, 7+5	8·1
Aug. 9	1815	3−1, 7+4	8·2
12	1818	3−1, 7+5	8·1
14	1820	3−1, 7+5	8·1
18	1824	3−2, 7+4	8·2
20	1826	3−3, 7+3	8·3
24	1830	3−4, 7+2	8·4
31	1837	3−4, 7+2	8·4
Sept. 2	1839	3−4, 7+1	8·5
10	1847	7+1	8·5
15	1852	=7	8·6
23	1860	7−1, 13+5	8·7
28	1865	7−1·5, 13+4·5	8·8
Oct. 1	1868	7−1·5, 13+4·5	8·8
4	1871	7−2, 13+4	8·8
7	1874	7−2·5, 13+4	8·8
13	1880	7−3, 13+3	8·9
Nov. 1	1899	7−4·5, 13+1	9·1

The Observations—continued.

Date.		Julian Date.	Comparisons.	Ded. Mag.
1918				
Nov.	7	1905	7 − 4, 13 + 1	9·1
	9	1907	7 − 5, 13 + 0·5	9·1
	13	1911	13 + 1·5	9·1
	15	1913	13 + 1 ; 16 + 3	9·1
	29	1927	17 − 1, 24 + 1	9·6
Dec.	7	1935	13 − 2·5, 17 + 1	9·4
	24	1952	23 − 1, 29 + 2 ; 22 − 1 ; 24 − 2	9·8
1919				
Jan.	9	1968	19 − 5·5, 34 + 2	10·1
Jan.	22	1981	= 28	9·9
Feb.	8	1998	23 − 4, 27 + 4	10·1
Mar.	2	2020	34 − 1, 37 + 1	10·4
	16	2034	17 − 0·5, 19 + 0·5	9·6
	24	2042	13 − 1·5, 17 + 1·5; 16 + 2	9·3
Apr.	2	2051	7 − 4·5, 13 + 1	9·1
	18	2067	7 − 1	8·7
	20	2069	7 + 1	8·5

The Comparison Stars Used.

Hagen's Number.	Hagen's Magn.	Hagen's Number.	Hagen's Magn.
2	*7·0	21	9·7
3	8·0	22	9·7
4	8·2	23	9·7
6	8·5	24	9·7
7	8·6	28	9·9
8	8·7	29	10·0
11	9·1	33	10·1
12	9·2	34	10·3
13	9·2	37	10·5
16	9·4	39	10·5
17	9·5	40	10·6
18	9·5	42	10·8
19	9·6		

* The B.D. value as given in Hagen's Catalogue.

Ludgrove, New Barnet, Herts:
1919 *April* 29.

The Position Line in Navigation.

By Instructor-Lieut. W. M. Smart, M.A., D.Sc., R.N.

The Theory of Navigation may be briefly summarised in the statement that the corrected altitude of a heavenly body, combined with the knowledge of the G.M.T. of the observation, is sufficient to determine a small circle on the earth—the circle of position,—on which the observer must be situated at the time of the observation. S is the place on the earth at which the heavenly body is in the zenith at the time of observation, and its position is known. The dotted curve DJ is a small circle whose pole is S, and whose angular radius, SJ, is the true zenith distance of the observation; DJ is the circle of position. If C is the estimated position at the time of observation, then it is clear that only a small portion of the small circle in the immediate neighbourhood of C need be taken into practical consideration. The method employed most frequently for drawing the position line on the Mercator's chart is that of St. Hilaire. It consists in finding SC from the formula

$$\text{Haversine SC} = \text{Hav.}(\text{PS} \sim \text{PC}) + \sin \text{PS} \sin \text{PC} \text{ Hav. SPC} \quad (1)$$

all the quantities on the right being known.

CJ is thus found—known as the intercept. Also the azimuth A is found by calculation or from tables. C_1, D_1 will indicate points on the chart corresponding to C, D on the sphere. From C_1 a line is drawn making an angle A with the meridians, the intercept is marked off, and a line at right angles—the position line—is drawn. As the intercept is usually very small, the position line so drawn will satisfy all practical requirements. It is clear that, within certain limits, the position line so drawn is independent of the position C—a fact that is not sufficiently well appreciated. The position C can be chosen as best suits the particular observation, and it is the first object of this paper to consider within what limits this choice may be exercised, without introducing errors of practical importance into the position, on the chart, of the position line. The advantages of choosing a position C, in the practical application of navigation, will be apparent subsequently. The single position line is alone considered in this paper.

1. In fig. 1, C will represent the chosen position, and D the actual position, of the ship, for the moment supposed known. The co-ordinates of points on the sphere will be referred to the meridian PS and the equator. The declination of the body will be denoted by Δ. The co-ordinates of C are H, L, and of D $H + \Delta H$, $L + \Delta L$. ΔI will denote the intercept CJ, and z the zenith distance SC found by calculation. Then $SD = z + \Delta I$. The angles are supposed to be expressed in minutes of arc. In particular, ΔI will express the number of nautical miles between C and the position circle. A will be supposed calculated or otherwise found. Then from C_1 a straight line will be drawn on

the chart making an angle A with the meridians, a distance representing ΔI (in nautical miles) measured along it from C_1, and a straight line drawn at right angles through the point so found. This line will not in general coincide with the true position line: it will be shown what the magnitude of the error will be.

2. Let (α, β), $(\alpha+\Delta\alpha, \beta+\Delta\beta)$ be the co-ordinates of two consecutive points on the position circle DJ.

Then

$$\cos(z+\Delta I) = \sin\Delta\sin\beta + \cos\Delta\cos\beta\cos a$$

and

$$\cos(z+\Delta I) = \sin\Delta\sin(\beta+\Delta\beta) + \cos\Delta\cos(\beta+\Delta\beta)\cos(\alpha+\Delta\alpha),$$

from which

$$\frac{d\beta}{d\alpha} = \cos\beta\tan A \quad . \quad . \quad . \quad . \quad (2)$$

On the chart let the axis of y be the part of the meridian of S

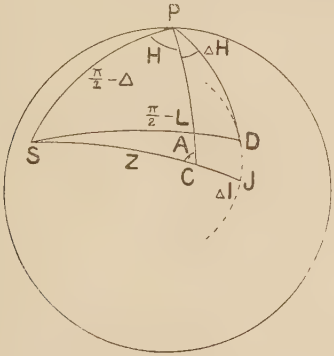


Fig. 1.

from the equator northwards, and the axis of x the equator drawn eastwards.

Then, if (x, y) represents a point (α, β),

$$x = \alpha$$
$$y = \log_e \tan\left(\frac{\pi}{4} + \frac{\beta}{2}\right).$$

The chart is a conformal representation of the surface of the earth. α is expressed in terms of ′ of longitude, and y in terms of the same unit. From the latter equation

$$\left.\begin{aligned}\cos\beta &= \operatorname{sech} y\\ \sin\beta &= \tanh y\end{aligned}\right\} \quad . \quad . \quad . \quad (3)$$
$$\frac{dy}{d\beta} = \sec\beta.$$

From (2)

$$\left(\frac{dy}{dx}\right)_J = \left(\tan A\right)_J.$$

In actual practice $(\tan A)_C$ is taken instead of $(\tan A)_J$, so that the slope of the position line is simply $\tan A$. When C is at a considerable distance from the circle of position (as is contemplated in this paper), A may differ from the angle of slope of the true position line by as much as 2° under certain conditions—the slope of the true position line being $\tan A'$, where A′ is the azimuth at D. The procedure explained in § 1 will thus involve certain errors, and it is to be shown that these will be of negligible practical importance.

3. In fig. 2, C_1A_1 is drawn, making an angle A with the meridians, *i.e.* with the axis of y. A_1E_1 is the position line, C_1A_1 represents the intercept ΔI. The unit of measurement on the chart is 1′ of longitude, and ΔI is expressed in nautical miles. Let K denote the distance C_1A_1 measured in ′ of longitude. The relation between K and ΔI is easily obtained.

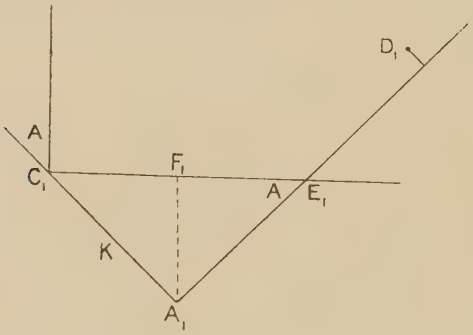


FIG. 2.

Let A_1 represent a place of latitude $L - \Delta L'$, and let its y-co-ordinate be $Y - \Delta Y'$, the co-ordinates of C_1 being (X,Y). It is necessary to obtain expansions up to the second order of the small quantities $\Delta L'$, ΔI, etc.

Then, from (3)

$$\tanh (Y - \Delta Y') = \sin (L - \Delta L'),$$

from which we derive

$$\Delta Y' = \Delta L' \sec L - \frac{(\Delta L')^2}{2} \sec L \tan L \,.\, \sin 1' \quad . \quad (4)$$

Now

$$A_1F_1 = \Delta Y' = K \cos A.$$

Also, since C_1A_1 represents the rhumb-line on the sphere drawn from C, of constant angle A,

$$\Delta L' = \Delta I \cos A.$$

Hence

$$K = \Delta I \sec L - \frac{(\Delta I)^2}{2} \sec L \tan L \cos A \,.\, \sin 1' \quad . \quad (5)$$

From the preceding it is obvious that, in general, A_1 does not represent J on the chart, inasmuch as CJ is a great circle and C_1A_1 represents a rhumb-line.

4. Let $(H + \Delta H, L + \Delta L)$ be the co-ordinates of D, and $(X + \Delta X, Y + \Delta Y)$ of D_1.

From SPD, SPC we have

$$\cos(z + \Delta I) = \sin(L + \Delta L)\sin\Delta + \cos(L + \Delta L)\cos\Delta\cos(H + \Delta H)$$

and

$$\cos z = \sin L \sin\Delta + \cos L \cos\Delta \cos H,$$

from which

$$\begin{aligned}\Delta I = {} & \Delta H \sin A \cos L - \Delta L \cos A \\ & + \tfrac{1}{2}\sin 1' \cot z[\Delta H \cos A \cos L + \Delta L \sin A]^2 \\ & - \Delta H \Delta L \sin 1' \sin A \sin L \\ & - \frac{(\Delta H)^2}{2}\cos A \sin L \cos L \sin 1' \quad . \quad . \quad . \quad (6)\end{aligned}$$

5. If p denotes the length of the perpendicular from D_1 to A_1E_1, measured in minutes of longitude, then p will provide the necessary criterion in considering the errors arising from the choice of the position C.

Let the equation of A_1E_1 be

$$x \tan A + b = y \quad . \quad . \quad . \quad . \quad (7)$$

Then

$$p = \frac{(X + \Delta X)\tan A + b - (Y + \Delta Y)}{\sec A}.$$

The co-ordinates of E_1 are $(X + K \operatorname{cosec} A, Y)$.

Hence

$$X \tan A + b - Y = -K \sec A.$$

Therefore

$$p = \Delta X \sin A - K - \Delta Y \cos A \quad . \quad . \quad . \quad (8)$$

Now

$$\Delta X = \Delta H,$$

and from (4),

$$\Delta Y = \Delta L \sec L + \frac{(\Delta L)^2}{2}\sec L \tan L \sin 1'.$$

Using (5), (6) in (8), we find that the first-order terms disappear. The result is

$$\begin{aligned}& p \operatorname{cosec} 1' \cos L \\ = {} & -\tfrac{1}{2}\cot z\,[\Delta H \cos A \cos L + \Delta L \sin A]^2 \\ & + \frac{(\Delta H)^2}{2}\sin L \cos L \cos A(1 + \sin^2 A) \\ & + \Delta L \,.\, \Delta H \sin^3 A \sin L \\ & - \frac{(\Delta L)^2}{2}\sin^2 A \cos A \tan L \quad . \quad . \quad . \quad . \quad (9)\end{aligned}$$

It is more convenient to express the perpendicular in nautical miles. Let it be P. Then at D_1 the scale of the chart is $\sec(L+\Delta L)$, so that

$$p = P \sec(L+\Delta L).$$

It will be sufficient to take $p = P \sec L$, which will give P to the second order in ΔL, ΔH.

Thus P cosec 1′ is given by the right side of (9).

6. It will now be assumed that the largest numerical value ΔH, ΔL can have will be 30′. The largest possible values P can attain, subject to these conditions, will now be investigated. There will be two cases—

(*a*) ΔL, ΔH, having the same sign and numerical value 30′.

(*b*) ΔL, ΔH, having opposite signs and numerical value 30′.

P, P_1 will denote the perpendiculars in the two cases.

(*a*)
$$P = -M \cot z + N \quad . \quad . \quad . \quad . \quad (10)$$

where

$$M = \frac{2}{15}[\cos A \cos L + \sin A]^2$$

$$N = \frac{2}{15} \tan L \cos A \,[\cos^2 L - \sin^2 L \sin^2 A] + \frac{4}{15} \sin^3 A \sin L,$$

in which

$$\frac{2}{15} = \frac{(30)^2}{2} \sin 1' \text{ with sufficient accuracy.}$$

(*b*)
$$P_1 = -M_1 \cot z + N_1 \quad . \quad . \quad . \quad . \quad (11)$$

where

$$M_1 = \frac{2}{15}(\cos A \cos L - \sin A)^2$$

$$N_1 = \frac{2}{15} \tan L \cos A \,[\cos^2 L - \sin^2 L \sin^2 A] - \frac{4}{15} \sin^3 A \sin L$$

If $M(\alpha)$ denotes the value of M when the azimuth is α, we have the relations, useful in the subsequent calculations, viz.,

$$M_1(180° - A) = M(A).$$
$$N_1(180° - A) = -N(A).$$

It will be noticed in (10) and (11) that the factor which will exercise, in general, the greatest effect on P and P_1 is cot *z*. In

practice, the observation of heavenly bodies with an altitude greater than 60° is avoided whenever possible. Burdwood's *Azimuth Tables*, which are in daily use at sea, do not give azimuths of bodies of altitude greater than 60°. In the following tables, however, maximum altitudes of 75° will be considered, in addition to those of 60°, *i.e.* when

$$\cot z \equiv \cot 15^\circ = 3{\cdot}73,$$
$$\cot z \equiv \cot 30^\circ = 1{\cdot}73.$$

In a very few instances, P, calculated from the maximum cot z, does not represent the maximum error; *e.g.*

In lat. 60°, azimuth 90°, N = 0·2 and $-\mathrm{M}\cot z = -0{\cdot}3$ (for $z = 30^\circ$), so that P = −0·1. The maximum P is clearly 0·2, corresponding to altitude 0°. It is this error, viz. 0·2, that is tabulated.

The errors are tabulated below for latitudes 0°, 30°, 45°, 60°, and for azimuths 0°, 30°, 45°, 60°, 90°, 120°, 135°, 150°, 160°.

TABLES I.

Maximum Altitude 75°.

	Lat. 0°				Lat. 30°			
Azimuth.	−M cot z.	N.	P.	P_1.	−M cot z.	N.	P.	P_1.
0	−0·5	N = 0	−0·5	−0·5	−0·4	0	−0·4	−0·3
30	−0·9		−0·9	−0·1	−0·8	0·1	−0·7	0·0
45	−1·0		−1·0	−0·0	−0·8	0·1	−0·7	0·0
60	−0·9		−0·9	−0·1	−0·8	0·1	−0·7	−0·1
90	−0·5		−0·5	−0·5	−0·4	0·1	−0·3	−0·5
120	−0·1		−0·1	−0·9	−0·0	0·1	0·1	−0·9
135	−0·0		−0·0	−1·0	−0·0	0·0	0·0	−0·9
150	−0·1		−0·1	−0·9	−0·0	0·0	0·0	−0·9
180	−0·5		−0·5	−0·5	−0·4	−0·1	−·05	−0·4

	Lat. 45°				Lat. 60°			
Azimuth.	−M cot z.	N.	P.	P_1.	−M cot z.	N.	P.	P_1.
0	−0·3	0·1	−0·2	−0·1	−0·1	0·1	0·1	−0·2
30	−0·6	0·1	−0·5	0·0	−0·5	0·1	−0·4	0·0
45	−0·7	0·1	−0·6	0·0	−0·5	0·1	−0·4	−0·2
60	−0·7	0·1	−0·6	−0·2	−0·6	0·1	−0·5	−0·3
90	−0·5	0·2	−0·3	−0·7	−0·5	0·2	−0·3	−0·7
120	−0·1	0·1	0·1	−0·8	−0·2	0·1	−0·1	−0·7
135	0·0	0·0	0·0	−0·8	0·1	0·1	0·1	−0·6
150	0·0	0·0	0·0	−0·7	0·0	0·0	0·0	−0·6
180	−0·2	−0·1	−0·3	−0·4	−0·2	0·0	−0·2	−0·2

TABLES II.

Maximum Altitude 60°.

	Lat. 0°.				Lat. 30°.			
Azimuth.	− M cot *z*.	N.	P.	P_1.	− M cot *z*.	N.	P.	P_1.
0	−0·3		−0·3	−0·3	−0·2	0	−0·2	−0·1
30	−0·5		−0·5	0·0	−0·4	0·1	−0·3	0·0
45	−0·5		−0·5	0·0	−0·4	0·1	−0·3	0·0
60	−0·5		−0·5	0·0	−0·4	0·1	−0·3	−0·1
90	−0·3	N=0	−0·3	−0·3	−0·2	0·1	−0·1	−0·3
120	−0·0		−0·0	−0·5	−0·0	0·1	0·1	−0·5
135	−0·0		−0·0	−0·5	−0·0	0·0	0·0	−0·5
150	−0·0		−0·0	−0·5	−0·0	0·0	0·0	−0·5
180	−0·3		−0·3	−0·3	−0·2	−0·1	−0·1	−0·2

	Lat. 45°.				Lat. 60°.			
Azimuth.	− M cot *z*.	N.	P.	P_1.	− M cot *z*.	N.	P.	P_1.
0	−0·1	0·1	+0·1	0·0	−0·0	0·1	0·1	−0·1
30	−0·3	0·1	−0·2	0·0	−0·3	0·1	−0·2	0·0
45	−0·4	0·1	−0·3	0·0	−0·3	0·1	−0·2	−0·1
60	−0·4	0·1	−0·3	−0·1	−0·3	0·1	−0·2	−0·2
90	−0·3	0·2	0·2	−0·5	−0·3	0·2	0·2	−0·5
120	−0·0	0·1	0·1	−0·5	−0·1	0·1	0·1	−0·4
135	−0·0	0·0	0·0	−0·5	0·0	0·1	0·1	−0·4
150	−0·0	0·0	0·0	−0·4	0·0	0·0	0·0	−0·4
180	−0·1	−0·1	−0·2	−0·2	−0·1	0·0	−0·1	−0·1

An inspection of the tables shows at a glance that when the maximum altitude is 60° the maximum error will not, on any occasion, exceed 0·5 (in nautical miles). When the maximum altitude is 75°, the maximum error would be, under the most unfavourable circumstances, 1 nautical mile. In the case of altitudes of 60°, the probable error would be of no practical importance. In the case of altitudes of 75°, the probable error would not exceed 0·3 nautical mile. Considering the practical difficulties of navigation, we may conclude that, for all altitudes up to 60° at least, the position line, so obtained, gives a sufficiently satisfactory result.

7. It is interesting to find the difference between the azimuth at the chosen position C and that at D.

We have

$$\cos \Delta = \cos z \sin \mathrm{L} + \sin z \cos \mathrm{L} \cos \mathrm{A},$$

and

$$\cos \Delta = \cos (z + \Delta I) \sin (L + \Delta L) + \sin (z + \Delta I) \cos (L + \Delta L) \cos (A + \Delta A),$$

from which

$$\Delta A = \Delta L \sin A \tan L - \Delta H \sin L \cot z + \Delta H \cos L \cos A \quad (12)$$

There will be two cases as before, viz.

when ΔL and ΔH have the same sign and numerical value 30′, and

when ΔL, ΔH have opposite signs and numerical value 30′.

ΔA, ΔA_1 will denote the errors in the two cases.

$\Delta A(75^\circ)$ will denote the error when the maximum altitude is 75°.

ΔA is tabulated in ′ for the same latitudes and azimuths as in Tables I. and II., and is given, irrespective of sign, to the nearest 5′.

TABLES III.

(*a*) *Latitude* 0°.

Azimuth.	ΔA (for all altitudes).
0	30
30	30
45	20
60	15
90	0
120	15
135	20
150	30
180	30

(*b*) *Latitude* 30°.

Azimuth.	$\Delta A(75^\circ)$.	$\Delta A(60^\circ)$.	$\Delta A_1(75^\circ)$.	$\Delta A_1(60^\circ)$.
0	30	0	30	0
30	0	0	40	15
45	25	0	50	20
60	20	0	60	30
90	40	30	75	45
120	50	20	85	55
135	60	30	85	60
150	95	40	90	60
180	80	50	80	50

TABLES III.—*continued.*

(*c*) *Latitude* 45°.

Azimuth.	$\Delta A(75°)$.	$\Delta A(60°)$.	$\Delta A_1(75°)$.	$\Delta A_1(60°)$.
0	60	15	60	15
30	45	0	70	25
45	50	5	90	45
60	50	0	100	55
90	50	20	110	65
120	70	25	120	75
135	80	40	120	75
150	85	40	115	70
180	105	60	105	60

(*d*) *Latitude* 60°.

Azimuth.	$\Delta A(75°)$.	$\Delta A(60°)$.	$\Delta A_1(75°)$.	$\Delta A_1(60°)$.
0	80	30	80	30
30	55	0	110	60
45	55	5	125	70
60	45	10	140	85
90	45	5	150	100
120	60	10	155	105
135	80	25	140	90
150	85	30	135	85
180	110	50	110	60

As far as the single position line is concerned, the effect of introducing an error of, say, 2° into the azimuth would be of small practical importance. Such an error would only occur under the most unfavourable circumstances. The probable error would certainly not exceed $\frac{1}{2}$°—a negligible amount so far as the drawing of the position line is concerned.

8. Throughout it has been assumed that D is the actual position of the ship: at the time of observation, it is, of course, unknown. With the improved instruments and machinery of modern ships the navigator can, using also his experience of the effect of weather on his ship, determine an estimated position, R, which, he is moderately certain, lies within a very few miles of his true position. An intercept of 10 miles, by the St. Hilaire method, is regarded as uncommon, and would be the result of faulty reckoning or the absence of observations for a considerable interval. If we take the estimated position to be in the immediate neighbourhood of the true position, and consider that ΔL, ΔH refer to the chosen position,

C, and the estimated position, R, we can see that the results in Tables I. and II. will indicate, as near as practical requirements demand, the order of the greatest errors that can possibly result from the employment of C in calculating the position line.

The method of choosing C will be best illustrated by an example. If the latitude of R is 51° 37′ N., the latitude of C will be chosen to be 52° N. If the hour angle for the heavenly body for R is $3^h\ 41^m\ 24^s$, the longitude of C will be chosen so that the hour angle for C is $3^h\ 40^m$, *i.e.* C is $1^m\ 24^s$ west of R. In this example $\Delta L = 23' : \Delta H = 21'$, and the errors for altitudes 75° and 60° are $\frac{8}{15}$ of those in Tables I. and II.

This is the principle involved in Ball's *Altitude Tables*, which give the altitudes for degrees of latitude and declination and every 4 mins. in hour angle. Interpolation for declination is, of

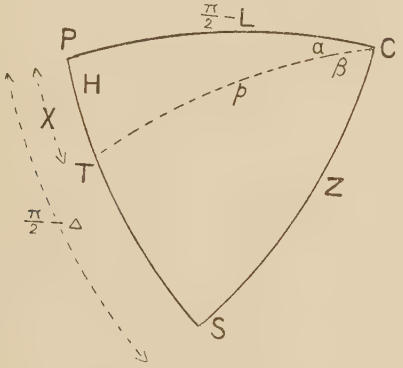


FIG. 3.

course, necessary. These tables (in two volumes) are bulky and expensive. The method indicated in the next paragraph is almost as rapid, and depends on the use of simple tables, consisting of about 30 pages, from which the necessary elements could be picked out very quickly.

9. Draw CT perpendicular to PS.

Then

$$\tan X = \cos H \cot L \quad . \quad . \quad . \quad (13)$$

and

$$\sin p = \sin H \cos L \quad . \quad . \quad . \quad (14)$$

Also

$$\cos z = \cos p \sin (X \pm \Delta)$$

or

$$\sin (\text{alt.}) = \cos p \sin (X \pm \Delta) \quad . \quad . \quad . \quad (15)$$

From (13) X is derived for every degree of latitude and 4 mins. of hour angle.

From (14) p is calculated and $Y \equiv \log \cos p$ is tabulated adjacent to X for the corresponding latitude and hour angle.

In (15) the + sign is taken when latitude and declination have the same names and the heavenly body is within 6 hours of the meridian, *i.e.* when

$$0 < \text{H.A.} < 6^h$$

or

$$18^h < \text{H.A} < 24^h$$

The − sign is taken when the body is more than 6^h from the meridian, *i.e.* when

$$6^h < \text{H.A.} < 18^h.$$

In this latter case latitude and declination must be of the same name.

The procedure for finding the altitude for the position C would be as follows :—

$X = 32^\circ 14'{\cdot}3$ (from the tables)
$\Delta = 10^\circ 17'{\cdot}8$ (say)

$X+\Delta = 42^\circ 32'{\cdot}1$	log sin	9·82997	
	Y	9·83625	(say, − from tables with same entry as for X)
	log sin (alt.)	9·66622	

and so the calculated altitude is $27^\circ\ 37'{\cdot}5$.

10. It will be observed that the tabulated quantities X, Y are independent of the declination. Logarithmic tables are consulted twice only, and the possibilities of making errors in the calculation of the altitude are almost entirely eliminated. Formula (1) is in direct contrast, as it requires no less than six references to logarithmic tables.

At sea, Burdwood's *Azimuth Tables* are in general use; interpolation for latitude, hour angle, and declination is usually necessary. In the method described, interpolation would be reduced to that for declination—an extremely simple process in practice.

The method is extremely accurate for observations near the meridian, as reference to Tables I., II. will show. The ex-meridian method at present in use requires specially constructed tables, and has no advantage in brevity over the method described.

Again, observations of the Pole Star are reduced, to obtain latitude, by means of the Pole Star tables in the *Nautical Almanac*. The latitude could be found almost as expeditiously by the method described. In the *Nautical Almanac* (abridged), the R.A. and declination of Polaris are given every three months. If these were given every month, the data would be quite sufficient to enable the latitude to be determined. The position C does not require to be plotted in the case of Pole Star observations.

The advantages of the method are that *all* observations for position line can be reduced by its means, without reference to such special tables as those for Pole Star and ex-meridian observations, that the tables required could be printed in 30 pages, and

that in the application of the method the possibility of making arithmetical errors is extremely small.

11. It is obvious that simple tables, not depending on the declination, can be constructed from which the azimuth could be readily obtained. From fig. 3,

$$\sin \alpha = \sin X \sec L \quad . \quad . \quad . \quad (16)$$

and

$$\cot \beta = \sin p \tan (\Delta + X) \quad . \quad . \quad . \quad (17)$$

α and log sin p could be tabulated for each degree of latitude and every 4 mins. of hour angle. B is obtained from (17) very simply, and the azimuth is $\alpha + \beta$ if the body observed is less than 6^h from the meridian, and $\beta - \alpha$ if it is more than 6^h from the meridian. Such tables would not occupy more than 30 pages, as in the case of the tables for calculating the altitude.

On a Long Period (about 240 Years) in Chinese Earthquake Records. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In a recent paper the Catalogue of Chinese Earthquakes compiled by Shinobu Hirota was analysed for the fifteen-month periodicity. At the end of the paper attention was drawn to a long period of 79 years which presented itself during the work. To investigate this and other possible periods the Fourier sequence was calculated for a series of 180 terms, each representing the total number of earthquakes recorded in 105 months in the Catalogue.

The results for the first 30 divisors were as in Table I.:—

TABLE I.

Fourier Sequence of Chinese Earthquakes.

Divisor.	Period. y	S.	C.	R².	Divisor.	Period.	S.	C.	R².
1	1575	−32	+ 3	1033	15	105	− 8	− 1	65
2	788	−22	−13	653	16	98	+ 2	− 2	8
3	525	+ 9	− 2	85	17	93	+ 3	+ 4	25
4	394	+10	+ 8	164	18	88	− 4	+ 2	20
5	315	+10	+ 3	109	19	83	− 1	− 1	2
6	262	+ 1	+22	485	20	79	+10	+ 1	101
7	225	− 1	− 5	26	21	75	− 1	+ 5	26
8	197	+ 6	+ 3	45	22	72	− 2	+11	125
9	175	0	+11	121	23	69	+ 6	− 5	61
10	158	− 8	−10	164	24	66	0	− 5	25
11	143	+12	− 9	225	25	63	+ 4	+ 2	20
12	131	+ 2	+13	173	26	61	+ 3	+ 8	73
13	121	− 3	+10	109	27	58	0	+ 2	4
14	112	− 1	+ 2	5	28	56			

2. The records are more numerous towards the end of the series than at the beginning, which accounts for the large value of $R^2 = S^2 + C^2$ for divisors 1 and 2. But this spurious effect dies down in the next few terms and cannot account for the large value of R^2 at divisor 6. The present note is chiefly confined to an examination of this apparent periodicity near 262 years: but we may glance at other large values of R^2. That which led to the present analysis is represented by divisor 20, period 79 years, $R^2 = 101$, and we see that there are also large values at divisors 22 and 26: these three terms 20, 22, and 26 being accompanied by large terms at 10, 11, and 13, while the large term at divisor 12 is probably connected with that at divisor 6, which we proceed to investigate, deferring consideration of the others for the present.

3. The first step is to arrange the observations in as nearly as possible the correct period, which is not necessarily represented by the exact divisor 6. The change of sign in both S and C between divisors 6 and 7 indicates that the true period is shorter than 262·5 years, and after several preliminary trials it was decided to adopt for discussion a period of $27\frac{1}{2}$ times the unit of 105 months, for which totals had been formed, *i.e.* 240·6 years. It is therefore simpler to return to the original Catalogue and form totals for every twenty years as in Table II., where the first number represents the years A.D. 1–20 and so on, the mean date being given by the sum of the figures in the top row and in the first column.

TABLE II.

Date.	0.	240.	480.	720.	960.	1200.	1440.	Total.
10	2	5	1	3	1	11	5	28
30	0	7	1	1	2	3	22	36
50	1	28	5	4	17	3	38	96
70	1	11	5	30	8	2	47	104
90	5	5	3	5	12	7	14	51
110	16	11	3	6	14	35	20	105
130	15	11	2	6	15	33	27	109
150	13	10	3	8	1	36	28	99
170	11	6	9	4	10	26	24	90
190	2	2	2	0	6	4	40	56
210	0	1	6	6	12	7	12	44
230	5	3	4	2	10	14	1	39
Sums	71	100	44	75	108	181	278	857

4. The totals in the last column show a considerable variation, but are clearly a good deal influenced by some large numbers of two kinds:—

(*a*) Those in the last two or three columns are due to an increase in the records assignable to greater facilities of communication. This influence can, however, be reduced or avoided by separate analysis of the seven columns.

(*b*) Some exceptional numbers in the earlier columns, such

as 28 for date 240 + 50 = 290, must, however, be treated in some special way. They are due to a number of earthquakes near together, possibly due to the same source. Instead of attempting a critical emendation of the records, the plan was adopted of substituting for the numbers in any column their order of magnitude, 1 to 12, in that column: thus the first column arranged in order of magnitude is

0 0 1 1 2 2 5 5 11 13 15 16

to which the new values

1 1 3 3 5 5 7 7 9 10 11 12

were assigned: and so for the other columns. The influence of exceptional numbers is thus considerably reduced, and the table now stands as in Table III.

TABLE III.

	0.	240.	480.	720.	960.	1200.	1440.	Total.
10	5	4	1	4	1	7	2	24
30	1	7	1	2	3	2	6	22
50	3	12	9	5	12	2	10	53
70	3	10	9	12	5	1	12	52
90	7	4	6	7	8	5	4	41
110	12	10	6	9	10	11	5	63
130	11	10	3	9	11	10	8	62
150	10	8	6	11	1	12	9	57
170	9	6	12	5	6	9	7	54
190	5	2	3	1	4	4	11	30
210	1	1	11	9	8	5	3	38
230	7	3	8	3	6	8	1	36
Sums	74	77	75	77	75	76	78	532

5. Analysing now the columns of both Tables II. and III., and distinguishing the results by suffixes 2 and 3, we get the following values for S and C, the coefficients of the sine and cosine and for R, θ deduced from $R^2 = S^2 + C^2$, $\tan\theta = S/C$.

TABLE IV.

Harmonic Analysis of Tables II. and III.

Date.	S_2.	C_2.	R_2.	θ_2.	$\theta_2-\theta_0$.	S_3.	C_3.	R_3.	θ_3.	$\theta_3-\theta_0$.
10	−1·0	− 6·8	6·9	189°	+30°	−0·3	−4·0	4·0	185°	+15°
250	+5·8	− 1·0	5·9	100	−59	+3·2	−1·8	3·7	118	−52
490	−0·7	− 0·3	0·8	247	+88	−0·7	−0·7	1·0	225	+55
730	+4·7	− 2·0	5·1	113	−46	+1·3	−2·8	3·0	154	−16
970	+1·8	− 2·0	2·7	138	−21	+1·5	−1·5	2·2	135	−35
1210	−4·7	−13·5	14·3	199	+40	−2·2	−3·0	3·7	216	+46
1450	+4·3	− 6·3	7·6	146	−13	+0·8	−2·0	2·2	157	−13
Mean	+1·7	− 4·5		159	±42	+0·5	−2·3		167	±33

The date in the first column is that of the first term in Tables II. and III., for which the angular variable is zero. The phases θ are fairly accordant, and the accordance of θ_3 is better than that of θ_2, as shown by the reduction of mean of errors from ± 42 to ± 33.

6. It does not seem possible at present to get a much closer estimate of the actual period. If we take the means of S and C for the first three rows and for the last three, and deduce the angles ϕ from $\tan\phi = S/C$, we find a slight increase, indicating that the adopted period of 240 years is too short. On the other

	S_2.	C_2.	ϕ_2.	$\theta_2-\theta_0$.	S_3.	C_3.	ϕ_3.	$\theta_3-\theta_0$.
First 3	+1·4	−2·7	153°	+19°	+0·7	−2·2	162°	+6°
Last 3	+0·5	−7·3	176	+ 2	0·0	−2·2	180°	− 1

hand, if we take the mean values of $\theta - \theta_0$, we get a slight decrement, indicating less than 240 years. The former method is to be preferred, and shows a larger change, so that the period may be put at (say) 243 years rather than 239. But, adopting the round figures 240 years, the maxima may be put at the dates

A.D. $840 \pm 240n$.

7. It was natural to look for some quite independent confirmation of this periodicity, and my first thought was to examine the records of Nile floods, which had been under intermittent consideration for some years, having been first put into my hands by Mr. J. I. Craig of the Egyptian Survey. Some of the readings are still under scrutiny, but any revision is not likely to affect the main results of Table V.

TABLE V.

Mean Readings of Nile Floods for Twenty Years in cm.

	Maxima.				Minima.			
A.D.	640.	880.	1120.	Mean.	640.	880.	1120.	Mean.
10	914	904	962	927	264	300	373	312
30	913	898	953	921	267	302	306	292
50	869	909	946	908	252	260	306	273
70	925	894	952	924	269	245	306	273
90	890	873	902	888	212	233	228	224
110	918	896	914	909	183	204	227	205
130	871	895	939	902	128	251	267	215
150	879	886	956	907	162	247	325	245
170	904	932	927	921	251	294	248	264
190	876	903	927	902	221	258	190	223
210	889	915	938	914	238	323	248	270
230	932	955	989	959	317	380	300	332

Analysing the columns harmonically, we get for the three periods of 240 years :—

TABLE VI.

Harmonic Analysis of Table V.

Dates.	Maxima.	Minima.
650–890	$11{\cdot}4\cos(\theta - 24^\circ)$	$59{\cdot}4\cos(\theta - 2^\circ)$
890–1130	$19{\cdot}9\cos(\theta - 312^\circ)$	$56{\cdot}6\cos(\theta - 326^\circ)$
1130–1370	$19{\cdot}4\cos(\theta - 346^\circ)$	$36{\cdot}3\cos(\theta - 28^\circ)$
Mean	$15{\cdot}2\cos(\theta - 341^\circ)$	$46{\cdot}0\cos(\theta - 355^\circ)$

8. The periodicity is thus clearly marked, especially in the minima. Again, it would be difficult to assign a more precise value to the length of period. The mean maximum for both maximum and minimum readings may be put at dates $645 \pm 240n$: or, to give actual dates,

A.D. 405, 645, 885, 1125, 1365, 1605, 1845,

which we may compare with the earthquakes

A.D. 360, 600, 840, 1080, 1320, 1560, 1800.

There is, of course, no reason why the maxima should actually coincide. The main point is the confirmation of periodicity, which seems quite good.

9. We now turn to Milne's *Catalogue of Destructive Earthquakes*, A.D. 7 *to* 1899. The heterogeneous nature of this material is emphasised by Milne in the introduction, and it is not surprising that a periodicity which is clearly marked in the more homogeneous Chinese records should be masked by this heterogeneity, or by the rapid increase in volume of the material as the world became explored and civilised. But the periodicity should still be discernible, and apparently it is so.

10. Let us first arrange Milne's whole material without critical selection of any kind, in groups of sixty years, using our Chinese information merely to choose the initial date, so that each group of sixty years may be symmetrically distributed round a maximum (such as A.D. 600), a minimum (A.D. 720), or the two nodes (A.D. 660 and 780).

TABLE VII.

Milne's Catalogue of Destructive Earthquakes.

Mean Date.	N.	log N.	Diff.	Less ·08.	Mean Date.	N.	log N.	Diff.	Less ·08.
60	7	0·84			(960)	(21)	(1·32)		
			+ ·16					+ ·22	+ 14
120	10	1·00			1020	35	1·54		
			− ·40					− ·02	− 10
180	4	0·60			1080	33	1·52		
			+ ·30					+ ·20	+ 12
240	8	0·90			1140	52	1·72		
			+ ·18					+ ·09	+ 1
300	12	1·08			1200	65	1·81		
			− ·13					− ·05	− 13
360	9	0·95			1260	58	1·76		
			− ·05					+ ·18	+ 10
420	8	0·90			1320	88	1·94		
			+ ·05					− ·02	− 10
480	9	0·95			1380	84	1·92		
			− ·11					+ ·05	− 3
540	7	0·84			1440	94	1·97		
			·00					+ ·20	+ 12
600	7	0·84			1500	147	2·17		
			+ ·34					− ·04	− 12
660	15	1·18			1560	136	2·13		
			+ ·12	+ 4				+ ·16	+ 8
720	20	1·30			1620	197	2·29		
			− 04	− 12				+ ·08	0
780	18	1·26			1680	234	2·37		
			+ ·28	+ 20				+ ·20	+ 12
840	35	1·54			1740	372	2·57		
			− ·20	28				+ ·16	+ 8
900	22	1·34			1800	540	2·73		
			− ·02	− 10				+ ·47	
960	21	1·32			1860	1579	3·20		

11. The first column contains the mean date of the sixty years counted; the second the total number (N) of earthquakes in the Catalogue; the third log N; and the fourth the differences of log N. Now there is a definite change in N after the first few centuries, which we reserve for separate consideration. Up to A.D. 600 there is no sensible advance in N or log N; but after A.D. 660 there is a more or less regular increase of 0·08 in log N per sixty years, until we come to the nineteenth century, when clearly a change of a different order takes place. Subtracting 0·08 from all the differences between these dates, let us arrange the residuals in sets of four as in Table VIII.

TABLE VIII.

Residuals of Table VII. in sets of four.

Residuals.				Middle—Ends.
+ 4	− 12	+ 20	− 28	+ 32
− 10	+ 14	− 10	+ 12	+ 2
+ 1	− 13	+ 10	− 10	+ 6
− 3	+ 12	− 12	+ 8	− 5
0	+ 12	+ 8		+ 12
− ·016	+ ·026	+ ·032	− ·045	

12. There are two positive means and two negative. Further, since the series begins with a node followed by a minimum, the signs are arranged in the right order—a drop, followed by two rises, then two drops, and so on. Further, if in each set of four we subtract the outside terms from the inside, as shown in the column "middle—ends," we get positive results in four cases out of five (supplying in the last row the 8 from the row above for the missing term). But it must be admitted that the indications are very precarious compared with those we obtained from the Chinese records, and the steady increase in volume of material seems scarcely a sufficient cause for the masking of the effect in this way. The mean difference in log N between a node and a max. or min. is only ·030, corresponding to about 7 per cent. This should be increased a little to compensate for taking sixty years together; but the corrected value is only 8 per cent.

13. Returning to the early centuries, the few earthquakes recorded are in general of great intensity (marked III. by Milne), but a few others have been added. The reasons for these additions are somewhat miscellaneous, and it seems better to confine attention to those marked III. The counts of these in successive forty years (beginning with A.D. 0–40, which is designated by its middle epoch 20) are as in Table IX. The first set of 720 years shows very little variation, but if we calculate the first harmonic, its maximum falls at 90°, *i.e.* at date 260 + 60 = 320 A.D. The corresponding Chinese maximum is 365 A.D. The second set of 720 years lies within the main series already considered. Its maximum is at 144°, or date 980 + 96 = 1076, the corresponding Chinese date being 1080.

After 1420 even the earthquakes marked III. show a rapid increase which embarrasses the discussion. The discovery of America, for instance, introduced new sources of information.

TABLE IX.

Milne's Earthquakes of Intensity III.

Date.	20.	60.	100.	140.	180.	220.
0	1	3	2	4	2	0
240	2	3	2	4	2	3
480	2	4	1	1	3	3
Sum	7	10	5	9	7	6
720	8	6	6	8	3	2
960	6	3	5	6	7	6
1200	8	6	10	15	5	3
Sum	22	15	21	29	15	11

14. Summing up the evidence so far, we may say that the 240-year period is strongly marked in the homogeneous (though probably far from complete) Chinese records; in the Nile maxima

and minima, especially the later; and is faintly shown in the *Catalogue of Destructive Earthquakes*: the general inference being that the most noteworthy earthquakes show it best.

15. We now turn to the term of long period in the Moon's longitude, to which Newcomb, Cowell, and E. W. Brown have drawn special attention in recent years. The period has been assigned as—

363 years, coeff. 18″ by Cowell (*M.N.*, 65, 34),
273 years, coeff. 13″ by Newcomb (*M.N.*, 69, 165),
261 years, coeff. 11″ by Brown (*M.N.*, 73, 708),
232 years, coeff. 8″ by Brown (*M.N.*, 73, 710),

the last-named value being chiefly illustrative. The term has barely run through one period during modern observation, and its length is thus uncertain. Is 240 years possibly the true value?

16. The origin of the term is at present unknown; but Newcomb has already considered as a possibility "that the inequalities are only apparent, being perhaps due to fluctuations in the Earth's speed of rotation, and therefore in our measure of time." He is inclined to reject this possibility as not supported by the records of transits of Mercury; but if it still remains open, then it may be remarked—

(*a*) That changes in the speed of rotation may naturally be associated with changes in the Earth's size, keeping the moment of angular momentum the same.

(*b*) The Moon's longitude was apparently increasing in 1720 and decreasing in 1860: that is, our day was long in 1720 and short in 1860: or the rotation speed being at its smallest in 1720, the radius of the Earth was compensatingly at its largest, and was therefore shrinking from 1720 to 1860, when it was at its smallest. Now it was just during these years (1720–1860) that large earthquakes were most numerous, 1800 being a maximum according to the Chinese analysis (§ 6 or § 8 of this paper). If, then, contraction denotes a time of exceptional stress, and expansion a relief, we can understand the observed association of phenomena. A change of 30″ in 120 years, or say 1500 lunations, is of the order 1/65,000,000, corresponding to about 4 inches in the Earth's radius.

17. There is another astronomical period of about the length under consideration. In *Monthly Notices*, 74, 100, I showed that the Chinese records of sun-spots show special outbursts at the dates

299, —, 826, 1077, 1370, 1616,

which are there connected with encounters between Saturn and the Leonids every 265 or 266 years. Could this possibly be adjusted to a period of 240 years? It is easily seen that the discordance would steadily increase: and the evidence is distinctly against any such shortening of the period. On the other hand, the earthquake period cannot reasonably be lengthened to suit the sun-spots. The identification does not commend itself in any way.

Summary.

(*a*) The periodogram or Fourier sequence of the Chinese earthquakes 285–1800 shows a marked periodicity somewhere near 260 years (Table I.).

(*b*) On scrutiny the best value is identified as 240 years or rather more (Tables II., III., and IV.).

(*c*) The Nile Flood readings A.D. 640–1360 show the same periodicity, especially the readings of minima, which have a steady range of nearly one metre in this period (Tables V. and VI.).

(*d*) Milne's *Catalogue of Destructive Earthquakes*, A.D. 7 *to* 1899, shows the periodicity, but only faintly. There are, however, difficulties in making proper allowance for the heterogeneity of the material, and for its rapid growth in volume (Tables VII., VIII., and IX.).

(*e*) If the periodicity in the lunar longitude can be so short as 240 years, and can be identified with a variation in the Earth's rotation, associated with a small pulsation in radius, then there are fewer earthquakes when the radius is expanding than when it is contracting, the dates showing good correspondence so far as they go (§§ 15 and 16).